

COLLEGE ALGEBRA IN CONTEXT: AN INTERDISCIPLINARY APPROACH

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We have developed an algebra course based on real life applications from business, economics, biology, and the social sciences in a setting that connects mathematical content with the real world. Data analysis, modeling, and technology are woven into the course so that the approach is refreshing and interesting to the students. The course provides the algebraic skills and concepts for a core course, or for the future study of calculus, in an informal, less threatening, and more meaningful setting. The course was designed to provide the required algebra skills for students in the business and economics majors and in majors in the biological and social sciences. In fact, this course provides the algebraic background needed for success in all majors other than the physical sciences and engineering. Because fewer colleges are requiring finite mathematics, it becomes important for students to solve meaningful applications in the college algebra course that they are taking. It is more essential than ever that students learn when and how to apply mathematics.

Most examples and exercises in the course are applied rather than skill problems. Each topic is introduced with a motivational example that presents a real life setting for that topic. The problem in this example is then solved after the topic has been discussed. For some topics, students work in small groups to solve extended application problems and to provide a written report on the results and implications of their study. For some other topics, students are required to find appropriate real data in the literature or on the internet, to make a scatter plot, to determine the function type that best models the data, to create the best mathematical model for the data, and to solve problems using the model.

We have found that students are less intimidated if functions are discussed first in the context of real data tables and graphs rather than equations. For example, functional notation can be discussed easily with the following examples:

1. **Car Financing** If a couple finances an automobile costing \$35,000 by making equal monthly payments over 5 years at an interest rate of 10%, the balance owed at the end of t years after purchase is given in the Table 1 below.

Table 1

t (years)	$B(t)$ Balance (dollars)
1	29,321
2	23,047
3	16,115
4	8458.60
5	0

(Source: Sky Financial Mortgage Tables)

- What is the balance owed by the couple at the end of 3 years?
- What is $B(1)$ if we represent the balance at the end of t years as $B(t)$? Write a sentence that explains its meaning.
- At the end of what year does the couple owe \$23,047?
- What is t if $B(t) = 8458.60$?

2. **Working Age** The projected ratio of the working-age population (25 to 64 year-olds) to the elderly shown in Figure 1 below defines the ratio as a function of the year shown. If this function is defined as $y = f(t)$ where t is the year, use the graph to answer:

- What is the projected ratio of the working-age population to the elderly population in 1995?
- Estimate $f(2005)$ and write a sentence that explains its meaning.
- What is the domain of this function?
- Is the projected ratio of the working-age population to the elderly increasing or decreasing on the domain shown in the figure?

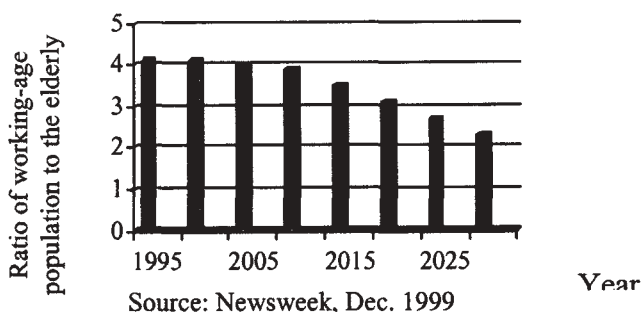


Figure 1

Note that problems include relevant questions about the models as well as mathematics questions.

When discussing linear functions, the connection between the slopes of the graphs and the rates of change of the functions permits us to ask much more interesting questions.

Average rate of change applications and discussions provide more opportunities to use the slope formula in a real context.

3. **Toyota Sales** The total Toyota hybrid vehicle sales for the years between 1997 and 2001 can be approximated by the model

$$S(x) = 2821x^3 - 75,653x^2 + 674,025x - 1,978,335,$$

where x is the number of years after 1990. Because $S(7) = 446$, there were 446 units sold in 1997; $S(9) = 16,505$, so the number sold in 1999 was 16,506.

(Source: www.toyota.com)

- Find the average rate of change of Toyota hybrid sales between 1997 and 1999.
- Interpret your answer to part a.
- What is the relationship between the slope of the secant line joining the points (7, 446) and (9, 16,505) and the answer to part a?

Writing linear equations from two data points can be more meaningful if the equation has a real basis after it is found.

4. **Men in the work force** The number of men in the work force (in millions) for selected decades from 1890 to 1990 is shown in Figure 2 below. The decade is defined by the year at the beginning of the decade and $g(t)$ is defined by the average number of men (in millions) in the workforce during the decade (indicated by the point on the graph within the decade). This data can be approximated by the linear model determined by the line connecting (1890, 18.1) and (1990, 68.5).

- Write the equation of the line connecting these two points to find a linear model for this data.
- Does this line appear to be a reasonable fit to the data points?
- How does the slope of this line compare with the average rate of change in the function during this period?

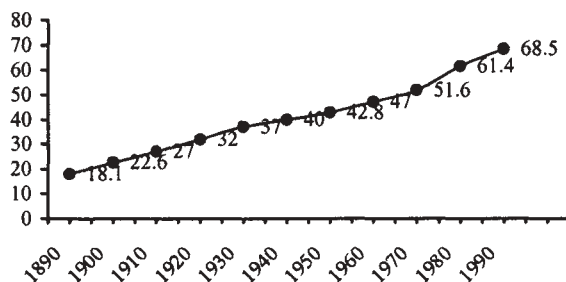


Figure 2

(Source: 1998 *World Almanac*)

Real data models can be used to expand the number and type of problems available for solution. These real data models are sometimes given to students and are sometimes developed by the students from the real data. So that students see that the mathematical models are really created from real data, it is important to have them use linear regression to create models.

5. **Internet Brokerage Accounts** The number of Internet brokerage accounts is given in Table 2 below.

- Write the linear equation that models the number of Internet brokerage accounts as a function of the number of years past 1990.
- According to the model, what is the annual increase in the number of accounts?
- Use the model to estimate when the number of accounts will be 20 million.
- What happened in 2001 that may make this model invalid for years after 2001?

Table 2

Year	1996	1997	1998	1999	2000	2001
Internet brokerage accounts (millions)	1.5	4.1	7.1	10.5	14.0	18.0

(Source: *Time*, May 31, 1999)

The vertex of a parabola is much more interesting if it is used to find a maximum or minimum value of a real data function.

6. **World population** A low projection scenario of world population for the years 1995 to 2150 by the United Nations is given by the function $y = -0.36x^2 + 38.52x + 5822.86$, where x is the number of years after 1990 and the world population is measured in millions of people.

- What is the x -coordinate of the vertex of the graph of this function?
- Use the x -coordinate of the vertex to graph this function.
- When does this model predict that the world population will have a maximum?
- What will be the world population in 2100 if the projections made using this model are accurate?

[Source: *World Population Prospects: 1998 Revision*, United Nations, 1999]

Graphical methods can be useful in solving applied quadratic equations. Unlike the skill problems that involve factoring, real data applications are usually not easily factorable. Finding the vertex of the graph of a quadratic function is a key step in the graphical solution of an applied quadratic equation.

An interesting application occurs when one solution is found easily with a graph, but the second one is not. The one solution can be used to find one factor, then the second factor and thus the second solution is easily found.

7. **Foreign-born population** The percent of the U.S. population that is foreign born can be modeled by the equation $y = 0.003x^2 - 0.42x + 19.8$, where x is the number of years since 1900.

- Graphically find one solution to $7.8 = 0.003x^2 - 0.42x + 19.8$.
- Use this information and factoring to find the year or years when the percent of the U.S. population that is foreign born is 7.8.

[Source: Bureau of the Census, U.S. Department of Commerce]

Students can use technology to fit quadratic and power functions to sets of data.

8. **Medicare Trust Fund balance** The year 1994 marked the 30th anniversary of Medicare. A 1994 pamphlet by Representative Lindsey O. Graham of the South Carolina 3rd Congressional District gave projections for the Medicare Trust Fund Balance as shown in Table 3 below.

- Representative Graham stated “The fact of the matter is that Medicare is going broke.” Use the data in the table to find when he predicted that this would happen.
- Find a quadratic model to fit the data. Find and interpret the vertex of the quadratic function.
- When does this model predict that the Medicare Trust Fund balance will be 0?
- The pamphlet said that the Medicare trustees stated: “... the present financing schedule for the hospital insurance program is sufficient to ensure the payment of benefits only over the next 7 years.” Does the function in part *b* confirm or refute the value 7 in this statement?

Table 3

Year	Medicare Trust Fund Balance (billions of dollars)	Year	Medicare Trust Fund Balance (billions of dollars)
1993	128	1999	98
1994	133	2000	72
1995	136	2001	37
1996	135	2002	-7
1997	129	2003	-61
1998	117	2004	-126

Exponential and power functions are also useful in modeling real data.

9. **Smokers** The percent of all U. S. persons 18 years of age and older that smoked in selected years from 1965 to 1995 is given in the Table 4 below.

- Graph the data.
- Can an exponential function be used to describe these data?
- Find an exponential function that models the data, using an input of the number of years past 1960.
- Find a power function that models the data, using an input of the number of years past 1960.
- Which model is the better fit for the data?
- Use the better model to graphically predict when the percent reaches 24%.
- Using the percent of smokers shown in the table above, find the logarithmic function that models the data.
- Is the logarithmic model a better fit than the exponential model?

Table 4

Year	1965	1974	1979	1983	1985	1987	1990	1991	1992	1993	1994	1995
Percent	42.3	37.2	33.5	32.2	30.0	28.7	25.4	25.4	26.4	25.0	25.5	24.7

[Source: Center for Disease Control, www.cdc.gov/nchs/datawh/statab/]