

## TANK TIME 2: STRUCTURED MAPLE PROJECTS FOR DIFFERENTIAL EQUATIONS

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### FACULTY DEVELOPMENT AT GEORGIA INSTITUTE OF TECHNOLOGY

The author was accepted into the Faculty Development Program at Georgia Tech for the Fall Semester 2000 and the Spring Semester 2001. My proposal was to write a number of structured Maple projects for differential equations. These projects would be “structured” in the sense that they would be written in the form of a Maple worksheet which would guide the student through completing the project. This guidance would allow for a minimal knowledge of Maple by the student since in many (although not all) cases, examples of the Maple syntax would be blended into the presentation and would precede the questions asked in the project. Other questions asked in these projects might, for example, produce a plot or an animation and require detailed explanations of these by the student. The idea was that students should learn some Maple, but not be overburdened by the Maple syntax since this would certainly take away from the main idea of using the technology to aid in understanding the mathematics.

### BEYOND GEORGIA TECH

Using my experience at Georgia Tech as a springboard, I have written a number of additional Maple projects for differential equations. The projects were originally written in Maple 9. I have also tested them in Maple 9.5 (the “classic worksheet” version and they ran just fine). The disk which I will be distributing (described below) was produced in Maple 9.5.

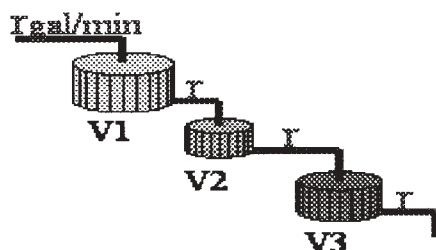
### THE DISK

During my presentation at ICTCMXV, I will be distributing a disk which contains a set of three structured Maple projects for differential equations, as well as complete solutions to each project. Tank Time Project I which is described below is not included on this disk, as it was presented at an earlier ICTCM meeting. This project can be found at the author’s web site. Although I retain the copyright to this material, I hope that those who receive the material will be able to use some of it in their differential

equations classes. I would ask, however, that you do not provide students copies of the project solutions or publish any of these solutions to the web. Each of these projects relates the the following problem:

### THE TANK TIME PROBLEM

Let's consider a system of three brine tanks containing  $V_1$ ,  $V_2$ , and  $V_3$  gallons of brine, respectively. A brine solution containing  $k$  pounds of salt per gallon flows into tank 1 at a rate of  $r$  gallons per minute, while mixed brine flows from tank 1 into tank 2, then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of  $r$  gallons per minute. Assume initially that tanks 1, 2 and 3 contain  $a$ ,  $b$ , and  $c$  pounds of salt respectively. Let  $x_i(t)$  represent the amount (in pounds) of salt in tank  $i$  at time  $t$ , for  $i = 1, 2, 3$ . We'll be interested in finding  $x_i(t)$ , of course. Let's see a graphic of this situation:



If we let:  $\frac{r}{V_1} = \alpha$ ,  $\frac{r}{V_2} = \beta$ ,  $\frac{r}{V_3} = \delta$ , and  $k r = p$ , then we obtain the system of three differential equations:

$$\begin{aligned} \frac{d}{dt}x_1(t) &= -\alpha x_1(t) && + p \\ \frac{d}{dt}x_2(t) &= \alpha x_1(t) - \beta x_2(t) \\ \frac{d}{dt}x_3(t) &= \beta x_2(t) - \delta x_3(t) \end{aligned}$$

A brief description of each of the these projects follows.

1. TANK TIME PROJECT I: We discuss some problems related to a cascade system of three brine tanks. This investigation leads immediately to a system of first-order linear equations. The theory of linear first-order systems and Maple provide us with a number of ways to solve these systems. One objective of this project is to show students how to apply some of these methods using Maple. We will actually use five different methods of solving the appropriate system of differential equations. The basic problem which will consider can be found in the text *Differential Equations and Linear Algebra* by Edwards and Penney as Example 2 on page 493; however, we state the problem in the more general form. First we discuss various ways to use Maple to help us solve the basic problem as it appears in Edwards and Penney. Later, we try to answer some deeper questions about a more general form of the problem using Maple to help us. The focus of the latter part of this project is obtaining a complete answer to the following question: at any given time  $t_0 > 0$  and any salt content  $c_0$  what conditions on  $t_0$  and  $c_0$  are necessary to make it possible for all three tanks have content  $c_0$  at time  $t_0$  by appropriate choices for the initial salt contents  $a$ ,  $b$ , and  $c$  of the three tanks?

2. TANK TIME PROJECT II: As in Tank Time Project I (see 1 above), we consider a cascade of three brine tanks. To be more specific, we consider the following problem: Consider a system of brine tanks containing  $V_1$ ,  $V_2$ , and  $V_3$  gallons of brine, respectively. A brine solution containing  $k$  pounds of salt per gallon flows into tank 1 at a rate of  $r$  gallons per minute, while mixed brine flows from tank 1 into tank 2, then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of  $r$  gallons per minute. Assume initially that tanks 1, 2 and 3 contain  $a$ ,  $b$ , and  $c$  pounds of salt respectively. Let  $x_i(t)$ ; represent the amount (in pounds) of salt in tank  $i$  at time  $t$ , for  $i = 1, 2, 3$ . Find  $x_i(t)$ , for  $i = 1, 2, 3$ . In this project, we fix the initial salt contents  $a$ ,  $b$ , and  $c$  of the three tanks and the flow rate  $r$ . We allow  $k$ ,  $V_1$ ,  $V_2$ , and  $V_3$  to vary and attempt to find conditions on these four variables which will give us 50 lb of salt in tank 1 after 1 minute, 100 lb of salt in tank 2 after 2 minutes and 150 lb of salt in tank 3 after 3 minutes. This project is completely independent of the project Tank Time Project I.

3. TANK TIME PROJECT III: In the solution of the problem described above in Tank Time Project II there are nine variables to contend with in our solution to the general problem:  $t$ ,  $r$ ,  $k$ ,  $a$ ,  $b$ ,  $c$ ,  $V_1$ ,  $V_2$ , and  $V_3$ . Clearly there are too many to deal with all at once! We will fix the values of seven of the nine variables and investigate the results one at a time. The variable  $t$  will never be fixed, so that means we will fix in turn seven of the variables  $r$ ,  $k$ ,  $a$ ,  $b$ ,  $c$ ,  $V_1$ ,  $V_2$ , and  $V_3$  while keeping the eighth one variable. The "fixed" values we will be using are:  $r = 10$  gallons/minute,  $k = 2$  pounds/gallon,  $a = 25$  pounds,  $b = 50$  pounds,  $c = 75$  pounds,  $V_1 = 100$  gallons,  $V_2 = 200$  gallons and  $V_3 = 300$  gallons. We'll be interested in seeing the effect on our solutions of changing that eighth variable. Specifically, in each instance we will be interested in finding the values of those variables and the times  $t$  for which the salt contents of two or more tanks are equal. The amount of analysis required to complete this investigation is too much for a single project. So we will begin the investigation here but it will require two additional projects to complete it. One of these additional projects is described below, but the final project in this series has yet to be written! In this project we consider first the problem of varying the flow rate  $r$ . After this analysis, we consider the problem of varying the concentration of the incoming brine  $k$ . This project is completely independent of the projects Tank Time Projects I and II.

4. TANK TIME PROJECT IV: In this project we consider the effects of varying the initial salt contents of the three tanks:  $a$ ,  $b$ , and  $c$ . See the description of Tank Time Project III above for further details. This project is independent of all of the other Tank Time projects.

#### THE WEB SITE

The web site given below contains a sample of the differential equations Maple projects as well as some additional Maple worksheets. Please feel free to access this material and to use whatever you find appropriate in your own classes. The copyright on this material is, however, retained by myself.

<http://www2.SPSU.edu/math/fadyn/index.html>

The material can also be found by beginning at the Southern Polytechnic State University home page and then navigating to Professor Fadyn's home page. The home page for Southern Polytechnic State University is located at: <http://www.spsu.edu/>