TEACHING MATHEMATICAL PROOF WITH TECHNOLOGY

Doug Ensley and Winston Crawley
Shippensburg University
Mathematics Department
Shippensburg, PA 17257
deensl@ship.edu

This NSF-funded project (DUE-0230755), currently in its second year, is focused on the development of pedagogically sound computer-based tools for teaching and learning mathematical proof.

Motivation

In order to evaluate the validity of proposed explanations, students must develop enough confidence in their reasoning abilities to question others' mathematical arguments as well as their own. In this way, they rely more on logic than on external authority to determine the soundness of the mathematical argument. (NCTM, pp. 345-346)

Even though, as Steen writes, "Nothing divides research mathematicians and mathematics educators from each other as do debates about the role of proof in school mathematics" (Steen, p. 275), the NCTM has long been an advocate of reasoning and proof in K-12 instruction. The NCTM *Principles and Standards for School Mathematics* specifically states that, "Reasoning and proof are not special activities reserved for special times or special topics in the curriculum but should be a natural, ongoing part of classroom discussions, no matter what topic is being studied." (NCTM, p. 342) Steen further suggests that, "The important question about proof may not be whether it is crucial to understanding the nature of mathematics as a deductive, logical science, but whether it helps students and teachers communicate mathematically." (Steen, p. 275)

Certainly for the teaching of mathematical proof to be successful in the K-12 curriculum, the teachers at that level should be confident about mathematical reasoning and communication in many forms. A teacher must be able to listen to a mathematical argument, in any form from the empirical to the precise, and give adequate feedback as to the argument's correctness and clarity. To do this, it is essential that teachers know how to read and write mathematical proofs in a variety of styles.

It is therefore an important time for mathematicians to supply curricula and methods for best preparing teachers to understand proof at many levels so that the objectives of these standards can be achieved. To accomplish this ambitious goal, this Educational Materials Development grant is currently developing computer-based activities to supplement an innovative discrete mathematics textbook, also co-authored by Doug Ensley and Winston Crawley, which is the first college course in which students at Shippensburg University encounter formal mathematical proof. The goals and objectives of the development of this material are as follows:

- To strengthen students' understanding of the logic of implicational ("if..., then...") statements.
- To teach students to read a formal proof as an interactive dialog.
- To develop new tools for teaching students how to write proofs.
- To contribute to the literature on teaching and learning mathematical proof.
- To train others to use the technology tools to implement the same ideas in other courses.

The nature of the material itself is fairly simple. It is based upon the basic idea that *students need to learn to understand logic and read proofs before they can write proofs.*

Applications under development

The following summarizes the actual types of activities being developed and currently being tested. The interested reader is directed to the website reference to try out the activities or for more information. (The website is organized into sections based on the Ensley/Crawley text, but the material may be used independent of that text.) At the 2004 ICTCM we emphasize material developed during the second year of the project – specifically, material for proof by contradiction and proof by induction.

CounterExamples. This is a collection of mathematical statements that a student must read critically and decide if each statement is true or false. If the statement is false, the student must provide a counterexample. This encourages students to construct their own understanding of truth or fallacy, and it establishes a point of view from which to write proofs of statements they believe are true.

ProofReader. To understand mathematical proof, one must be able to first effectively read mathematical proofs. Under this simple premise, we will develop a second *Flash* application in which students "trace" through a formal proof and respond to each statement therein. This process is not unlike the debugging process that is done in computer programming, but our main objective is for students to see mathematical proof as an interactive dialog between author and reader. Within this general format, we have "incorrect proofs" of false statements as well as "incorrect proofs" of true statements. This twist connects with the type of discovery practiced in the *CounterExample* exercises, and it also gives students a tool by which they can discover particular errors in a proposed proof.

Shuffled Proof. To further encourage students to read proofs for mathematical content and logical understanding, we have created some simple examples of proofs where the lines of the proof are presented in a scrambled order. The student must "drag" the statements into the correct order to complete the exercise. Figure 1 (next page) demonstrates a proof by contradiction that the student has begun to unscramble.

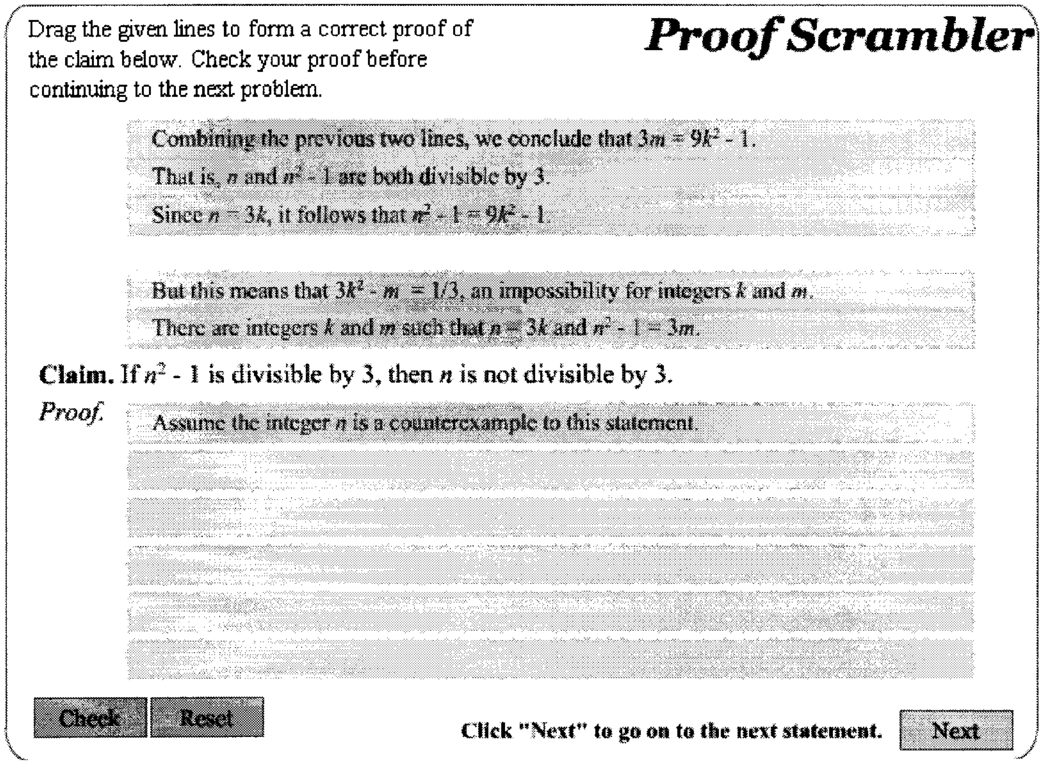

Figure 1 – Proof scrambler

Mathematical induction. "Proof by induction" is a fairly specialized technique that is very important in the discrete math course. Once again we take advantage of technology to help students create their own understanding of the process. The technology is used first to reinforce and enhance the students' ability to reason recursively, both for recursively defined sequences and for summations. In class, we use tabular layouts similar to these to help the students learn to calculate recursively:

n	Recursive formula $a_1 = 2,\ a_n = a_{n-1} + 2n$
1	2
2	$2 + 2 \cdot 2 = 2 + 4 = 6$
3	$6 + 2 \cdot 3 = 6 + 6 = 12$
4	$12 + 2 \cdot 4 = 12 + 8 = 20$
…	…….
30	$a_{29} + 2 \cdot 30 = \ldots = 930$
31	$a_{30} + 2 \cdot 31 =$ (do it)

n	Recursive formula $a_1 = 2,\ a_n = a_{n-1} + 2n$
1	2
2	$2 + 2 \cdot 2 = 2 + 4 = 6$
3	$6 + 2 \cdot 3 = 6 + 6 = 12$
4	$12 + 2 \cdot 4 = 12 + 8 = 20$
…	…….
m-1	$a_{m-2} + 2 \cdot (m-1) = \ldots = m^2 - \mathrm{m}$
m	$a_{m-1} + 2 \cdot m =$ (do it)

Later, we extend these ideas to verifying that a given recursive formula and a given closed formula describe the same sequence. In class, we use table layouts similar to the following, again beginning with the concrete:

n	Recursive formula $a_1 = 2$, $a_n = a_{n-1} + 2n$	Closed formula $a_n = n^2 + n$	Are they the same?
1	2	$1^2 + 1 = 1 + 1 = 2$	Yes
2	$2 + 2 \cdot 2 = 2 + 4 = 6$	$2^2 + 2 = 4 + 2 = 6$	Yes
3	$6 + 2 \cdot 3 = 6 + 6 = 12$	$3^2 + 3 = 9 + 3 = 12$	Yes
4	$12 + 2 \cdot 4 = 12 + 8 = 20$	$4^2 + 4 = 16 + 4 = 20$	Yes
...	....	....	....
30	$a_{29} + 2 \cdot 30 = \ldots$ $= 930$	$30^2 + 30 = 900 + 30$ $= 930$	Yes
31	$a_{30} + 2 \cdot 31 =$ (do it)	$31^2 + 31 =$ (do it)	???

and moving toward the abstract:

n	Recursive formula $a_1 = 2$, $a_n = a_{n-1} + 2n$	Closed formula $a_n = n^2 + n$	Are they the same?
1	2	$1^2 + 1 = 1 + 1 = 2$	Yes
2	$2 + 2 \cdot 2 = 2 + 4 = 6$	$2^2 + 2 = 4 + 2 = 6$	Yes
3	$6 + 2 \cdot 3 = 6 + 6 = 12$	$3^2 + 3 = 9 + 3 = 12$	Yes
4	$12 + 2 \cdot 4 = 12 + 8 = 20$	$4^2 + 4 = 16 + 4 = 20$	Yes
...	....	....	....
m-1	$a_{m-2} + 2 \cdot (m-1) = \ldots$ $= m^2 - \mathrm{m}$	$(m-1)^2 + (m-1) = \ldots$ $= m^2 - \mathrm{m}$	Yes
m	$a_{m-1} + 2 \cdot m =$ (do it)	$m^2 + m$	???

The technology provides an interactive setting for the student to work with this table-based visual model for recursive thinking and for proof by induction. Figure 2 (next page) illustrates the use of the technology for one of the concrete calculations.

Conclusion

We are very pleased with the preliminary results and we are looking forward to further development as well as continued research into the way students learn mathematical proof.

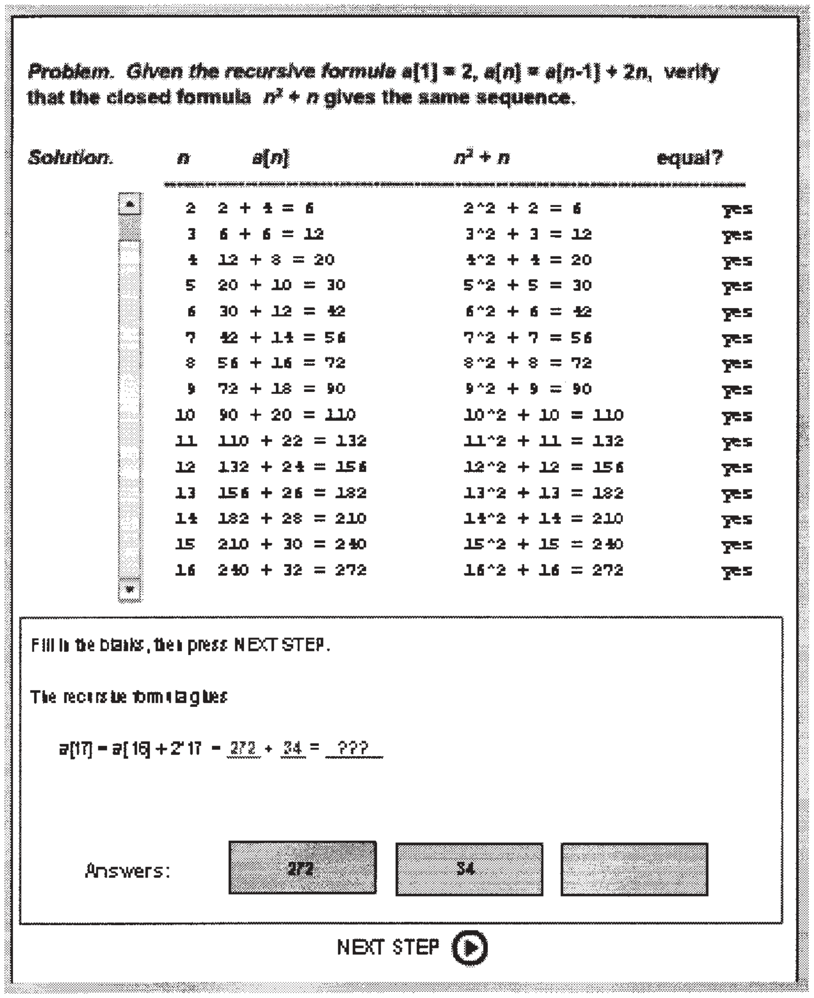

Figure 2 – Proof by induction

References

Ensley, D. E. and J. W. Crawley, Discrete Math course web site with links to *Flash* applications for teaching mathematical proof and developer's material (zipped). http://www.ship.edu/~deensl/DiscreteMath/

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