# TEACHING AND LEARNING BUSINESS CALCULUS THROUGH TECHNOLOGY

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Abstract

> The concepts of differentiation and integration are fundamental to understanding many mathematical connections with the natural sciences, the social sciences, and with business. To fully understand the mathematical concepts within the course, it is necessary that students experience them through observation and by having hands-on experience that involves multiple and interconnecting representations (symbolic, graphic, numerical and verbal) of the concepts. It is very difficult for a student to use a concept effectively if the concept is not fully understood. To fully understand mathematics concept, I believe, requires experiencing it in a variety of interconnecting ways. Technology holds the key for accurate and appealing representations that enables the student to make the mathematical connections both quickly and effectively.

During the session in New Orleans, I focused on specific mathematical concepts that are fundamental to a calculus course for business majors and presented ways in which students can best learn these concepts through in-class demonstrations by the instructor, integrated with student hands-on experiences using technology.

Technology that is commonly used in an applied calculus classrooms of today might include: hand-held devices for basic calculations, graphics representation of functions, viewing and analyzing numerical patterns from a table of values, and program applications of such activities as estimating integral values or approximating derivatives as limits of slopes of secants lines that approach tangency to the function; computers for analyzing and viewing numerical and graphical representation of such data as growth or decay values, and for using such dynamic software programs as *Derive* or *MathCad* to facilitate conceptual understandings through multiple representations of such fundamental concepts as instantaneous rate of change or integral sums. Ed Dubinsky and David Tall [**1**] indicated that students need experiences in developing relationships and making the connections among symbolic, graphic, numerical and verbal representations of these concepts. Many new texts include designated software for use for specific topics and chapter sections of the book. Some of the software is becoming quite good with dynamic software applications and high quality graphics. In addition to having access to the many applets available on the Internet, technology resources for teaching mathematics have become quite numerous.

Many classrooms are equipped with ceiling mounted video projects, have hard-wire or wireless Internet access, and in some cases have such devices as *Elmo's*, *Smartboards* and/or computer stations for each student. As a result, the instructional decision regarding the use of technology for teaching mathematics has become not so much whether or not to use technology but when to and when not to use it, and what is the most appropriate technology tool and application (from all that is available) for the specific topic or lesson planned.

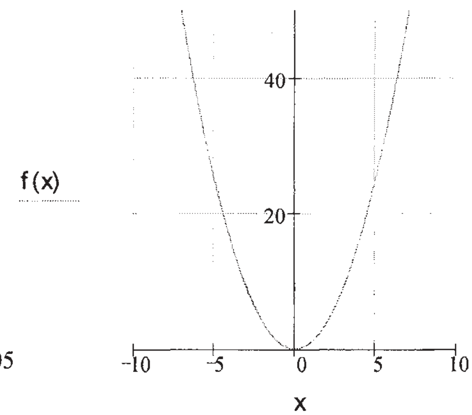
In each instructional setting, however, one needs to: (a) focus on the content and the instructional goals, before deciding the role of technology (Can technology enhance instruction or is it a barrier between the student and the mathematical ideas?), (b) look at the role of the students if technology is going to be used, (Are students to be active learners using the technology to gain insight into the mathematics concept or passively viewing the instructor's "screen?"), and (c) look at the role of the instructor (Is s/he "facilitating" student learning?). The most recent meta-analysis of research on the effects of technology on student outcomes suggest that students who use technology in their learning had positive gains in learning outcomes over students who learned without technology **[2]**.

To fully understand the mathematical concepts within a calculus course for business majors, it is necessary that students experience these concepts through passive observances of the instructor's work and active, hands-on experiences of their own that involve different yet equivalent representations of the fundamental mathematics related to business calculus. In any case, the proper instruction tool must be selected and used in a way so that the technology does not become a barrier (syntactically or otherwise) between the student and the concept(s) being addressed. The visualization that is possible with today's dynamic software enables the student to see and experience mathematical relations and concepts that were difficult to "show" or explain in days prior to technology. For example, to show an instantaneous rate of change as a limit of average rates of change as the denominator approaches zero, prior to technology, the instructor would draw several different secant lines on a board or transparency showing their limiting slope to be that of the slope of a tangent line. Now with dynamic software, a student can move a slider of the x-coordinate of a point on the curve (or just "dragging" the secant/tangent line itself) and new secants/tangents are drawn, the technology shows a much clearer picture of such concepts of limits and tangents.

**The following are several examples of calculus concepts in which students learn through hands-on use of technology**

1. Using software such as *Mathcad*, the students can easily discover the limiting value of the difference quotient representing the slope of the secant lines as they approach tangency to the function $f(x) = x^2$ at the point (5,25).

$f(x) := x^2$




h := 2.0 , 1.9 .. .1

h := 1 , .95 .. .05

h := .05 , .045 .. .005

| h | f(5+h) | $\frac{f(5+h) - h(5)}{h}$ |
|---|---|---|
| 0.05 | 25.502 | 10.050 |
| 0.045 | 25.452 | 10.045 |
| 0.040 | 25.402 | 10.040 |
| 0.035 | 25.351 | 10.035 |
| 0.030 | 25.301 | 10.030 |
| 0.025 | 25.251 | 10.025 |
| 0.020 | 25.200 | 10.020 |
| 0.015 | 25.150 | 10.-15 |
| 0.010 | 25.100 | 10.010 |
| 0.005 | 25.050 | 10.005 |

Figure 1: Exploration of Derivative at a Point using *Mathcad*

After explorations such as in Figure 1, students are asked to complete such statements as: "Hense the instantaneous rate of change of $f(x) = x^2$ at $x = 5$ is _____ ." Then eventually to more general statements as: "If $f(x) = ax^n$, then $f'(x) =$ ______."

The students also have hands-on experiences with technology to help them solve such problems as problems 2 and 3, below.

2. A salsa company has a cost function, $C(q) = 0.01q^3 - 0.6q^2 + 13q + 1000$ where q is the number of cases of salsa produced and C(q) is in dollars. If 100 cases are produced, find the average cost per case.

Having experienced the graphical representation of "average cost" (AC) as the slope of a line from the origin to the cost curve and then using their familiar concept of average as a total sum divided by the number of item, students can easily establish the following.

$$C(q) = 0.01q^3 - 0.6q^2 + 13q + 1000$$

$$C(q) := 0.01q^3 - 0.6q^2 + 13q + 1000$$

$$AC(q) := 0.01q^2 - 0.6q + 13 + \frac{1000}{q}$$

$$AC(100) := 63$$

Figure 2: Calculation for Average Cost, using *Mathcad*

The students enjoy using the computer for graphing, making tables, and doing various calculations, as well as the high quality of the print outs of their work.

3. At a price of $8 per ticket, a musical theater group can fill every seat in the theater, which has a capacity of 1500. For every additional dollar charged, the number of people buying tickets decreases by 75. What ticket price maximizes revenue?

Students are to determine the revenue function, take its derivative, set the derivative equal to zero and then solve it to determine critical points. Since the revenue function is a quadratic with a negative leading coefficient, they are aware that the graph is concave down and thus has a critical point that is the maximum for the function. To encourage multiple representations, to check their answers and to support the visual learning of many students, I encourage (sometimes require) them to also graph and/or make a table after determining the revenue function.

$$R = pq \qquad q = -75p + b$$

$$1500 = -75 \cdot 8 + b$$

$$2100 = b$$

$$\text{so,} \quad R(p) := p(-75p + 2100)$$

They then graph R(p) to find the maximum value of the revenue function and to reinforce their findings of R to be $14,700 which occurs for a ticket price of $14.

Figure 3: Calculation and Text of the Revenue Function, from *Mathcad*

Others will solve it numerically by making a table

$p := 8, 9 .. 18 \quad q := 1500, 1425 .. 750$

| p = ▪ | q = ▪ | pq = ▪ |
|---|---|---|
| 8 | 1500 | 12000 |
| 9 | 1425 | 12825 |
| 10 | 1350 | 13500 |
| 11 | 1275 | 14025 |
| 12 | 1200 | 14400 |
| 13 | 1125 | 14625 |
| 14 | 1050 | 14700 |
| 15 | 975 | 14625 |
| 16 | 900 | 14400 |
| 17 | 825 | 14025 |
| 18 | 750 | 13500 |

Figure 4: Table of the Revenue Function, from *Mathcad*

**Concluding Remarks**

A significant question that should also be briefly addresses is: "What Should be the Role of Technology in Teaching a Calculus Course to Business Majors?" I think that because there is so much technology available today to teaching and learning mathematics we need to be careful not to use too many different technology pieces (Internet, graphics calculators, specific computer software, etc.) in any one mathematics class. During one's college career hopefully the students will have had experience with a wide variety of technology in learning mathematics. In any case, the driving force behind the use of any piece of technology must be the specific mathematics topics and deciding what topics within the course can truly be enhanced through instruction with technology. Technology must not be a "barrier" but a bridge between the student and the mathematical concepts of the course.

**References**


1. E. Dubinsky and D. Tall, Advanced Mathematical Thinking and the Computer, in D Tall, ed., *Advanced Mathematical Thinking,* Kluwer Academic Publications, 1994, 231-243.

2. H, Waxman, M. Connell and J. Gray. A Quantitative Synthesis of Research on Effects of Teaching and Learning with Technology on Student Outcomes. Retrieved June 11, 2003 from www.ncrel.org/tech/effects.