CALCULUS LABS FOR BIOLOGY AND PRE-MED STUDENTS

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Introduction

In an effort to better prepare biology and pre-medical students for the increasing level of mathematical background needed for their future coursework, the Department of Mathematics at Benedictine University has begun to offer a two-semester calculus sequence for this audience. Although these two courses are offered at the same level of mathematical rigor as the traditional sequence for students majoring in mathematics, physics, and engineering, the content is specifically geared to meet the needs of the biology and pre-med students. In this paper, we discuss issues and considerations for the computer algebra laboratory projects designed for the calculus sequence for the biology and pre-med students. One of the most valuable aspects of these activities is the integration of mathematical and biological reasoning, which better prepares the students for future course work in the biological sciences and which motivates the students to understand the importance of mathematics in the biological sciences. Hopefully, this new sequence will encourage more biology majors to continue to take additional mathematics courses.

In addition to traditional calculus content, there are two main mathematical needs of the students who are either majoring in biology or are pursuing future careers in medicine. The first of these needs are developing the ability to read, represent, and interpret biologically significant data presented in numerical and graphical formats. This ability is essential for both the future researchers as well as the future medical professionals as most summary data in scientific and medical literature will be provided in such formats rather than in symbolic terms via equations and mathematical models. Thus approaching material through the "The Rule of Four" popularized by the Calculus Reform (Renewal) movement is natural. Even so, additional emphasis needs to be placed on graphical representation of data that are not commonly found in traditional calculus courses. These include semilog and log-linear plots and, for discrete population models, plots of offspring versus parents. (It is important to note that the University's Biology Department offers a biometry course. It can be easily argued that the statistics content is just as important, if not more important, than calculus content for the students under discussion. To eliminate overlap in an already tight schedule, the calculus sequence does not include probability and statistics content.)

The second need is the ability to understand, interpret, and apply basic mathematical (biological) models in terms of functions, rates of change, and simple difference and differential equations. Mathematical biological models frequently involve several quantities–some variable and some constant parameters. It is challenge for the students to understand the difference between independent quantities, dependent quantities, and constant parameters. Developing this skill will, in turn, help the students understand the biological significance of equations and formulas presented to them in their biology coursework. Additionally, understanding how to create and read basic biologically significant mathematical models can be a challenge for these students whose current background in biology is still limited. When creating models, the students not only can see the power of being able to express biological in mathematical terms but also can develop an understanding of assumptions required to construct a model and the limitations of the model. Applications are essential and traditional components of calculus courses, but the applications in the first two semester lean heavily toward physics and engineering. The attention to the modeling process is what makes these courses different and often more difficult and makes significant connections between mathematics and biology. Another benefit of this focus on modeling is that the successful student who pursues graduate work in biology or mathematical biology will better be able to approach biological questions with a greater array of mathematical tools.

Current Courses

The differences between the this sequence and the traditional sequence reflect the students needs described in the previous paragraphs. Specifically, the changes made for the biology sequence are the emphasis on biological examples and models, alternate representations of data, and an increased emphasis on data-driven problems. Although we do integrate the modeling aspect into the lecture portion of the course, many of the more complex problems that require students to use software to represent and interpret data and to construct and interpret models are structured as laboratory projects. The choice of topics and outlines for the biology calculus sequence also depends on some institutional considerations. At this time, we are only able to offer a single section of the biology calculus sequence, and not all students who are interested in taking the first semester of this course are able to register for it. We need to align the basic calculus content between to the two sequences to maintain reasonable flexibility so that students completing either Calculus I course would be able to take either Calculus II course. Moreover, our design is structured that students completing the two-semester biology sequence will be able to continue with Calculus III and Differential Equations in the following semesters.

The two Calculus I courses both provide a rigorous treatment of limits, continuity, differentiation and integration through the Fundamental Theorem of Calculus. (By "rigorous", we mean that formal definitions of limits and derivatives are discussed.) The biology version of Calculus I also includes discussion of semi-log and log-log plots and applications of regression prior to the actual calculus content. The most significant difference between the two versions of Calculus I is that the biology version precedes the traditional discussion of limits and continuity with a material on difference equations and discrete population models. The discussion of difference equations not only provides straightforward introduction to population models and differential equations but also provide nice examples for the study of other topics including limits (via sequences) and, in the multivariable setting, basic concepts in linear algebra, which are addressed in the second course. Applications of differentiation in the first course include the stability criterion of first-order difference equations and the graphical representation of stability through cobwebbing.

The Calculus II courses diverge considerably. Gone from the biology version is the traditional content of sequences and series. (The biology version still includes some techniques of integration, improper integrals, and Taylor polynomials.) The primary focus of the this course is to develop the tools needed to analyze systems of differential and difference equations. This requires the development of some basic linear algebra so that students can work with matrices, understand matrices as operators, and understand eigenvalues and eigenvectors. Basic tools of multivariable calculus are also needed including partial derivatives, the derivative matrix, and the concept of linearization. Where the traditional Calculus II course places more emphasis on theory (particularly with sequences and series), the biology version of the course emphasizes the use, development, and analysis of models using the mathematical tools. Even though the level of rigor in both versions is similar, the biology version takes on a much more applied tone.

Purpose of the Labs

The traditional calculus sequence at Benedictine University has included laboratory activities using the computer algebra system Derive for many years. Most recently, Calculus I courses include a weekly lab session and Calculus II and Calculus III assign six to seven laboratory projects requiring Derive. (As of the Spring 2005, the lab portion of Calculus I will be offered in a required but separate one-hour course with roughly the same amount of laboratory work as assigned previously in Calculus I.) Many of the goals we have for the students in traditional sections apply to the students in the biology sections as well. In particular, we expect students to learn how to use software to approach mathematical problems, learn calculus through discovery and investigation, work in a collaborative environment, and develop the ability to communicate mathematics in writing. Additional goals we have for laboratory

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projects geared toward the biology students are addressing calculus content via biologically oriented problems or topics and using problems that address mathematical models in biology. These laboratory activities are expected to approach the content via the Rule of Four.

The new Calculus I laboratory course will integrate labs with biological content. As these labs will emphasize modeling and the analysis of data, the activities will be beneficial to all Calculus I students. So that the students in the biology section of Calculus I receive enough examples of modeling and analysis of complex biologically oriented problems, several additional projects will be assigned. The primary difference between these projects is that they will be expected to be completed over a several week time frame rather than be completed during the laboratory session. The six or seven laboratory activities in the Calculus II course for the biology students follow the format of the additional projects in the biology version of Calculus I.

Examples of Topics for Labs and Projects

The projects and laboratory assignments do require the use of computer software. Most of our activities are designed for the computer algebra system Derive for which our university has a site license. As spreadsheets are widely available and are frequently used in scientific work, we use Excel in some of the projects. Topics in the first course that lend themselves to nicely to laboratory or extended homework projects include using regression to fit data and creating appropriate semilog or log-log plots, using spreadsheets to analyze recursively defined population models, analyzing bifurcation and chaotic behavior in the discrete logistic equation, analyzing equilibria and stability of difference equations, and determining an effective drug dosage using geometric series and exponential decay. Topics that lend themselves nicely for projects in the second course include the Leslie matrix model for age-structured populations ([2]), predator-prey models including the Lokta-Volterra model ([3], [7]), the Fitzhugh-Nagumo differential equation model for neuron activity ([1], [4]), and the Nicholson-Bailey host-parasitoid model as an example of a system of difference equations ([6]).

Future Directions

We are at the beginning of the process of developing the laboratory projects for the biology calculus sequence. We strive to make the courses and the associated activities relevant for the biology majors in the course sequence. To accomplish this goal, we have begun discussions with the Biology Department at Benedictine University so that they can provide us with examples, models, and real data related to their courses and research. These discussions will lead to coordination between the disciplines so that the students can better see connections between biology and mathematics: the power of mathematics in addressing biological questions and the increasingly quantitative nature biological work.

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