AN OPTIMAL COMPUTER FACILITY LOCATION

William P. Fox, Ph.D.
Francis Marion University
Department of Mathematics
PO Box 100547
Florence, SC 29501
wfox@fmarion.edu

William H. Richardson, Jr., Ph.D.
Francis Marion University
Department of Mathematics
PO Box 100547
Florence, SC 29501
wrichard@fmarion.edu

Consider the following problem. A small company is planning to install a central computer with cable links to five departments. According to the floor plan, the peripheral computers for the five departments will be situated using a 100 by 100 grid (measured in feet) as shown by the circles in Figure 1. The company wishes to locate the central computer so that the minimal amount of cable will be used to link the central computer to the five peripheral computers.

The five peripherals are located at integer-coordinate positions (15,60), (25,90), (60,75), (75,60), and (80,25). Cable may be strung over the ceiling panels in a straight line from a point above any peripheral to a point above the central computer, and it is not necessary to consider lengths of cable from a peripheral computer itself to a point above the ceiling panel located immediately over that computer. That is, we work only with lengths of cable strung over the ceiling panels. The central computer will be located at coordinates (m,n) where m and n are integers in the grid representing the office space. A solution to this minimization problem can be sought using concepts from multivariable calculus, nonlinear programming, computer programming, and integer programming. Since integer programming is not frequently offered as an undergraduate course in mathematics, this paper discusses solutions to the problem that could easily be used in teaching the first three courses.

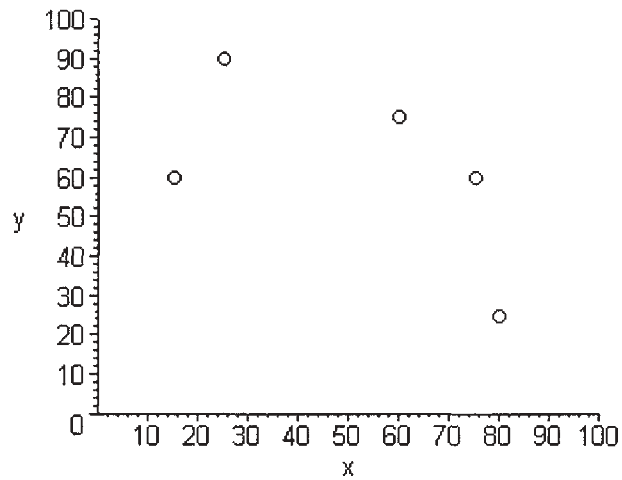

Figure 1. A grid showing locations of the five peripheral computers.

Multivariable Calculus

If the requirement that integer coordinates be found for the central computer's grid location is relaxed, multivariable calculus may be used to determine real-valued coordinates that minimize the number of feet of cable needed to connect all the computers. Subsequent rounding to integer coordinates may be reasonable. This must be checked. The function to be minimized represents the sum of the five distances from the peripheral computers to the central computer and is given by

$$f(x,y) = \sum_{i=1}^{5} \sqrt{(x - X_i)^2 + (y - Y_i)^2}$$

where for $i = 1,\ldots,5$, (X_i, Y_i) represents the coordinates describing the grid location of the i^{th} peripheral. A graph of $f(x,y)$ is shown in Figure 2. MAPLE was used to determine the first partials of the function *f*, which are shown using MAPLE notation in Figure 3. MAPLE's command *fsolve* was used to find coordinates $(x = 56.81841102, y = 68.07515715)$ that cause these two equations to vanish simultaneously. The determinant of the Hessian matrix of the objective function was determined to equal 0.01530846191. Evaluating $\frac{\partial^2 f}{\partial x^2}$ at the stationary point yielded a positive value (0.1416584426) as well, so the above-mentioned stationary point corresponds to a minimum value for the objective function (157.6634722 feet).

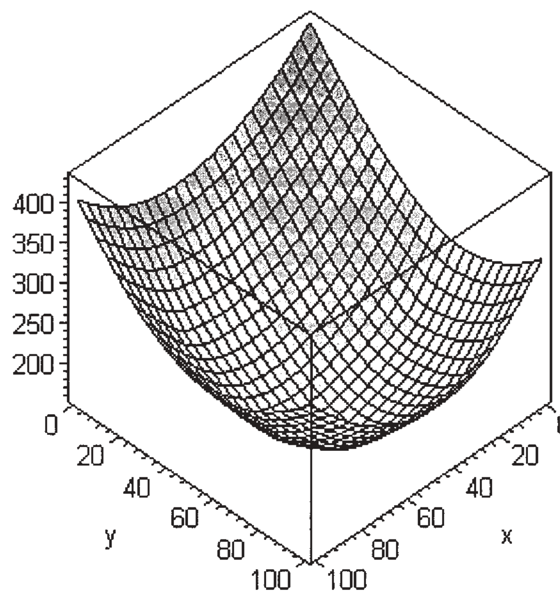

Figure 2. A plot of the objective function $f(x,y)$ representing the sum of five lengths of cable needed if the central computer is located at grid position (x,y).

Plotting the stationary point $(x = 56.81841102, y = 68.07515715)$ on a contour plot of the objective function revealed that it was reasonable to round these coordinates to the integer values $(x = 57, y = 68)$. Thus, using standard tools taken from multivariable calculus, it was found that grid location (57, 68) represents the optimal placement for the central computer. The resulting minimal feet of cable required is 157.6663224 (after rounding). The optimal location is depicted in Figure 4.

Nonlinear Programming

This optimization problem is an excellent candidate for methods taught in a nonlinear

$$fx := (x,y) \to \frac{1}{2}\frac{2x-30}{\sqrt{x^2-30x+3825+y^2-120y}} + \frac{1}{2}\frac{2x-50}{\sqrt{x^2-50x+8725+y^2-180y}} + \frac{1}{2}\frac{2x-120}{\sqrt{x^2-120x+9225+y^2-150y}} + \frac{1}{2}\frac{2x-150}{\sqrt{x^2-150x+9225+y^2-120y}} + \frac{1}{2}\frac{2x-160}{\sqrt{x^2-160x+7025+y^2-50y}}$$

$$fy := (x,y) \to \frac{1}{2}\frac{2y-120}{\sqrt{x^2-30x+3825+y^2-120y}} + \frac{1}{2}\frac{2y-180}{\sqrt{x^2-50x+8725+y^2-180y}} + \frac{1}{2}\frac{2y-150}{\sqrt{x^2-120x+9225+y^2-150y}} + \frac{1}{2}\frac{2y-120}{\sqrt{x^2-150x+9225+y^2-120y}} + \frac{1}{2}\frac{2y-50}{\sqrt{x^2-160x+7025+y^2-50y}}$$

Figure 3. $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ for the objective function f.

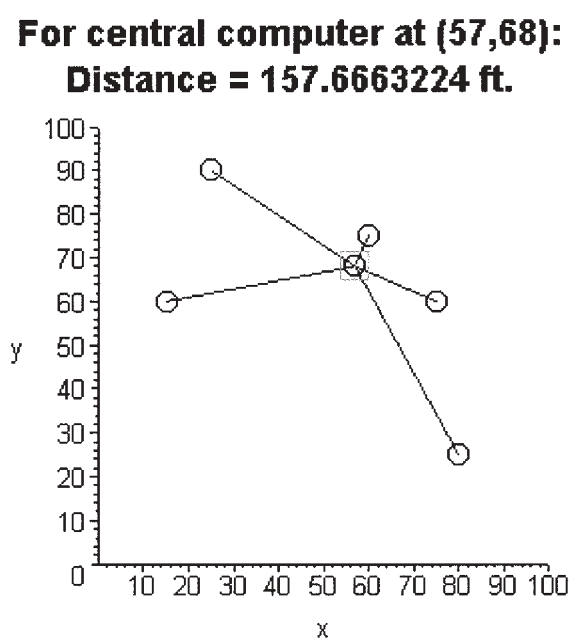

Figure 4. The optimal location for the central computer is grid position (57, 68). A minimum of 157.7 feet of cable is needed.

programming class. In previous work, Fox and Richardson[1] presented MAPLE procedures that implemented the well-known *Steepest Ascent* algorithm for maximizing a function. A MAPLE procedure that can be used to implement *Newton's Method* for optimizing a function was also presented. In this work, these two MAPLE procedures were used to determine the optimal placement for the central computer as discussed earlier.

First *Newton's Method* was applied using the grid location (40,50) as the initial location for the central computer. The procedure converged in four iterations yielding the approximate stationary point (56.8184427,68.07512727), which corresponded to an approximate minimum 157.6634722 feet of cable. Consistently positive eigenvalues for the associate Hessian matrices provided sufficient conditions to ensure the minimum value for the objective function. Next the *Steepest Ascent* MAPLE procedure was used starting with an initial grid placement of (0,0) for the central computer. The approximate stationary point (56.7180,67.92470) was found after three iterations with an approximate functional minimum 157.6652 feet. In both cases, plotting the stationary point on a contour plot of the objective function indicated that it was reasonable to round the coordinates of the stationary point to (57,68).

Computer Programming – An Exhaustive Search

While this optimization problem is suitable for a multivariable calculus classroom and a class in nonlinear programming, it has also been used as a problem in our *Introduction to Fortran 90 Programming* class at Francis Marion University. In this case, an exhaustive search of all possible grid locations (with integer coordinates) for the central computer was implemented using a *Fortran 90* program. The grid location corresponding to the minimum number of feet of cable needed to connect the computers was determined to be (57,68). The corresponding minimum number of feet was approximately 157.6663224. This search can be performed even more easily using a MAPLE procedure. Such a procedure is shown in Figure 5.

Summary

The optimization problem discussed here – determining the optimal grid location for a central computer to be connected to five peripherals – has been presented as a useful topic for a multivariable calculus class, a class in nonlinear programming, and a computer programming class. One of the authors intends to use it as a hallmark problem in the upcoming spring semester course in nonlinear programming at Francis Marion University. The other has used the problem successfully in a class in *Fortran 90* programming, and will later present the problem to a class in multivariable calculus at Francis Marion University. This optimization problem is suitable for a capstone course in mathematics since it offers students the opportunity to solve a practical problem using concepts learned in a variety of undergraduate courses. It is a good meld of technology and mathematics as it is sufficiently intractable to require computer software regardless of the solution method being used.

```
> restart:with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected

> Computer:=proc(m::integer,n::integer,f)
> local disthold,xhold,yhold,i,j,dist;
> disthold:=infinity;
> for i from 0 to m do
>     for j from 0 to n do
>        dist:=evalf(f(i,j));
>        if (dist < disthold) then
>            disthold:=dist;
>            xhold:=i;
>            yhold:=j;
>        end if;
>     end do;
> end do;
> printf("\n\nOptimal integer coordinates: (%d,%d).\nThe
cable length is %g feet.\n\n\n\n",xhold,yhold,disthold);
> end:
```

Figure 5. A MAPLE procedure used to determine the optimal grid location for the central computer using an exhaustive search of all possible integer-coordinate locations.

REFERENCES

1. Fox, William P. and Richardson, William H. "Multivariable Optimization When Calculus Fails: Gradient Search Methods and Newton's Method in Nonlinear Optimization Using MAPLE", Computers in Education Journal, Vol. XII(4), pages 2-11, October - December 2002.