MULTIPLE INTEGRATION - VISUALIZATION AND ANIMATION USING MAPLE

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Abstract

Students of engineering and science often find multiple integration difficult. This paper reports on a Maple "immersion mode" teaching (since 2002) of a calculus of vector functions course. Starting with an introduction to grad, div and curl and finishing with the integral theorems of Gauss and Stokes, the computer algebra system (CAS), Maple, is used for all presentation in the class and computation in the laboratory. No calculations are done by hand. All assignments are required to be done using Maple: they are submitted via the web and returned with marks and comments imbedded in the Maple file. The examination is held in the computer laboratory using Maple. Indeed, the examination "paper" is a Maple file (no hardcopy supplied!) and each student's examination response is submitted (as a Maple file) to their Digital Drop Box on the web. These are marked without any print out.

In particular, we discuss the use of the visualization (and animation) capabilities of Maple to introduce slicing diagrams to illustrate double (and triple) integration (as iterated or repeated integration). Path integrals are introduced in 2D to find the area of a wall built above a path. The volume under a surface is treated as volume of a "shed". Visualization is presented in the class and students are required, in the examination, to provide various plots (of the usual objects such as curves and surfaces) and also slicing diagrams for double integration.

Visualization in mathematics is not new: the ancient Greeks used visualization in a fundamental way as they did mathematics. What is new is the capability for visualization of modern computer software and calculators that provide new challenges to modify our curriculum and pedagogy to take advantage of these tools. We offer several courses, including the calculus of vector functions course discussed here, in a "Maple immersion" mode. Although considerable staff time is required to develop new teaching materials, students and staff regard these courses as successful and enjoyable.

Introduction

Almost all of our mathematics courses include a software usage component – usually the Computer Algebra System (CAS) Maple. Typically, the software is used in a "support" mode where, say, a weekly computer laboratory session supports the teaching in a manner that can be likened to a weekly practice class. Some assignments are completed using the software, but the examination is a traditional paper and pencil exam (with calculators allowed, but not computers). For innovative use of Maple and animations (including with first year courses in support mode) see [1] and the references therein.

We also conduct three courses (all in third year) in a Maple "immersion" mode: a Finite Element Methods course (which also includes a final section using commercial FEM software - see [2]); a Geometry of Surfaces course (which is classical differential geometry – see [3]); and a Vector Calculus Methods for Geospatial Scientists (the topic of this paper and of [4]). In these courses, lectures are presented using Maple, student assignments and examinations are completed using Maple.

Our Maple immersion mode of teaching has developed over time. Initially each lecture was matched with a computer lab session. As we developed our approach and Maple files, the Maple worksheets were used for presentation in lectures and students were encouraged to not take any notes. However the students wanted to have a hardcopy of the Maple files to annotate as they followed the lecture. In the second semester of 2004, the Vector Calculus course was conducted in a two-hour block in the lab rather than a lecture followed by a lab session. An opportunity was sought to break the teaching into mini-lectures of 20 to 30 minutes interspersed with some small task undertaken by the students. After a small break of 5 to 10 minutes, the rest of the time was used for a lab work sessions where students were encouraged to work cooperatively either on the "lecture" material or their assignment(s). The lecturer assisted with resolving any difficulties – including with the assignments.

There were several factors leading us to this teaching mode. One is our increasing experience with teaching in a Maple immersion mode and the conviction that this is the way of the future. We have been influenced by the pioneering work of Jerry Uhl and his co-workers; our approach is related (see [5]: Why (and how) I teach without long lectures). Further, we have been experimenting in lectures with a pause (of about 2 minutes) in mid lecture to take some account of the view that the maximum attention span is about 20 minutes. An encouragement to move away from the traditional lecture also came from a standard student feedback questionnaire (copy available on request) that was given to the same students in the last teaching week of the previous semester. This group of just less than 30 students took the Geometry of Surfaces course in Maple immersion mode in the previous semester. Besides the usual statements which students are invited to express agreement or disagreement on a 5 point Likert scale, there where a few questions asking for comments. In response to **"What would you change to improve the course?"** a few students suggested that "lectures scrapped and 2 hours of lab so we have a computer in front of us to follow. Very easy to get lost in lectures – thus we lose concentration".

We make two further comments about our teaching in two-hour blocks in the lab. All teaching and assessment materials were available before class on the web (we use

BlackBoard) and students downloaded the lecture file (which is provided without output – which reduces file size) and executed it. Thus the students had a live Maple file to play with as the "lecture" proceeded and they would be looking at different views of the plots of the objects (space curves, surfaces, etc). Also, no hardcopy of lectures (or assignments, past exam papers, exams …) were provided and unlike previous classes, there were no requests of the lecturer to provide hardcopies of lecture notes!

For a discussion and references on visualization, the use CAS as a pedagogical tool and on the Vector Calculus course, see [4]. This also discussed double integrals and slicing diagrams and their role in setting up the correct double integral and for plots of functions over irregular domains. The present paper is updated and complementary to [4].

Line integrals and area of walls

Most texts introduce line integrals by partitioning a space curve and introducing a work done integral. This misses an opportunity to provide a valuable and simpler example with some strong visualization, and so we start in 2D with a path in the x,y plane, build a wall of height f(x,y) above the path and ask "What is the area of the wall?".

We start with a discussion that we need to extend our integration of f(x) with respect to x as area under the curve. We use a teaching aide: a paper sheet cut to represent variable height, bent to represent a wall of variable height above a curved path. It is easy to see that the area of the wall is the same as the area under the curve when the path is straightened. The "unbending" of the wall is accomplished by using arc length along the path. Since the students have met arc length several times before, we briefly review arc length including a simple calculation. We provide a plot of a wall and pause our "lecture" to ask (with hints) the students to compute the area of a wall. This task is done fairly well.

As a preliminary to a discussion of path independence and potential, we build and plot together walls above different paths in 2D that have the same start point and end point.

Double integrals and volume of a shed

Double integrals are introduced as volume under a surface. The iterated integrals are represented (in the usual way) via slicing. We talk about slicing a loaf of bread and this is easy to represent in a Maple plot, see Figure 2 in [4]. The loaf of bread is assumed to have a rectangular base and vertical walls, so finding the volume by slicing and adding the volumes of the slices is conceptually and visually simple. We generalize to nonrectangular domains by considering a shed with a triangular base. A plot in 3D with slicing is provided, see [4]. It can be seen that the main effort required is to slice the domain of integration and so the slicing diagrams are introduced. An example is reproduced in top left hand side of Figure 1. Students are required to be able to produce these 2D slicing diagrams in Assignment 3 and the exam. From 2004, we also present an animation of the generation (by slicing) of the domain (three frames from the animation are shown in Fig. 1). Students are not asked to write such animations, but the Maple code is provided (and some do experiment with this). Once slicing is understood, we return to our walls by building a shed: slicing the domain is used to plot the roof and to set up the correct double integral for the shed's volume.

Since the writing of [4], we have discontinued using the Doubleint( ) command in the Student package which provides for typesetting. Besides needing to load the extra package, the value of the double integral was sometimes evaluated incorrectly. We now use

```
> Int(Int('F.n', r=S..` `), theta);
```

$$\int \int_{S} F\,.\,n\;dr\;d\theta$$

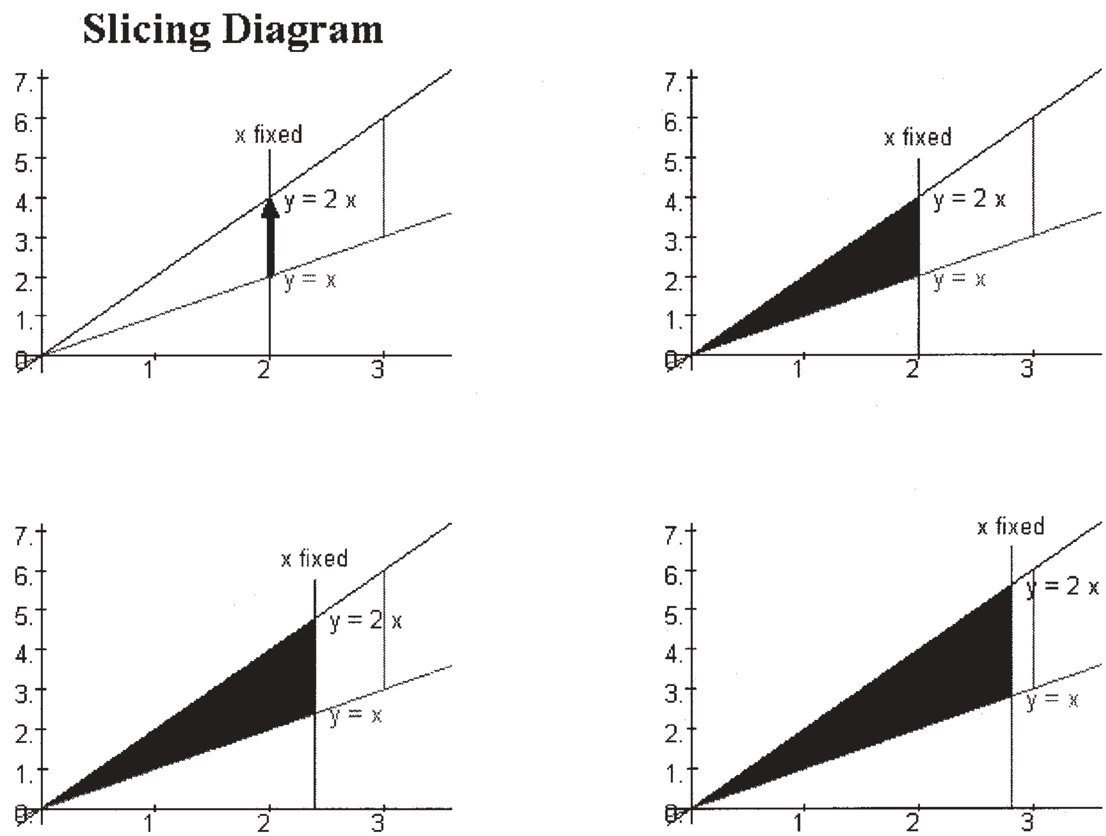

Figure 1. A slicing diagram for the 2D domain and 3 frames of an animation of slicing.

Triple integrals

Once slicing 2D domains of double integrals is understood, triple integrals are introduced, initially as volume integrals. The vital step is to be able to visualize the 3D object and the slicing. We start with a tetrahedron, using Cartesian coordinates x,y,z and then present examples where cylindrical and spherical polars are natural (and used). To assist with visualization we cut away part of the bounding surface, see Figure 2 (where the axes have been removed to improve the figure here – in class the axes are included).

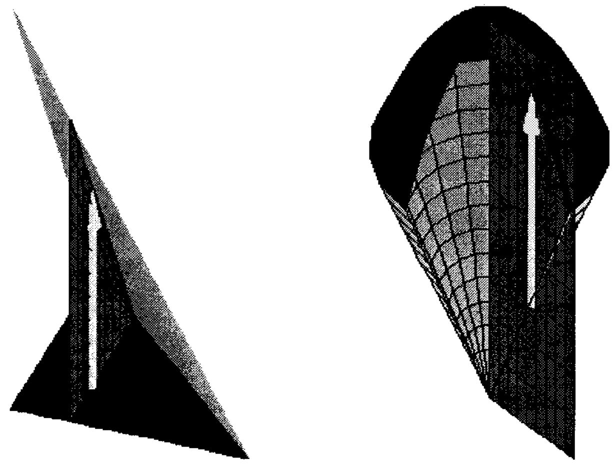

Figure 2. Slicing diagrams in 3D for volume of a tetrahedron and of an ice cream cone.

The tetrahedron in the first octant is bounded by the plane x+y+z =4. In Fig.2, the z axis is vertical and (to aid visualization) the xz plane is removed (from the "front") and the yz plane is also removed. The x fixed slice is a red triangle. These stack to form the 3D object. When x and y are fixed, only z varies to give the vertical yellow arrow - these stack to form the red triangle). Similarly, the ice cream cone (with a parabolic top and a cut-away) is sliced by a θ fixed red plane and vertical (θ, r fixed) yellow arrows. Students enjoy working with these Maple live and coloured plots. Thus they're able to successfully visualize the slicing of 2D or 3D objects to setup the correct multiple integral.

References

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