DERIVE 6: CAS AS A TOOL FOR TEACHING AND LEARNING MATHEMATICS

Lisa Townsley
Benedictine University
5700 College Rd, Lisle IL 60532
Ltownsley@ben.edu

The following paper is a synopsis of a talk and demonstration given at ICTCM-17, October 30, 2004, in New Orleans. The conference emphasizes the appropriate use of technology for collegiate teaching and learning: the recent upgrade of DERIVE to version 6.0 (6.1 now) makes it one of the premier computer algebra systems (hereafter spelled CAS) to use in the education arena. This introduction to the new (and some older) features of DERIVE will focus on classroom use, in lecture/discussion and student laboratory experimentation. More information can be found at the web addresses for Texas Instruments http://education.ti.com/us/software.html (where you can download a free trial copy) and the DERIVE-Europe site: http://www.derive-europe.com .

The heritage of DERIVE the program begins with muMath, produced by Albert Rich and David Stoutemyer in 1979. The focus of the program was to provide symbolic computing power on small computers. Then in 1988, DERIVE for DOS was introduced. The program required only 512K bytes of memory to operate, which allowed the author to implement a networked laboratory of XT machines with DERIVE. A major overhaul of the program came with the transition to a Windows interface in 1996. Therese Shelby was brought in as a third author to assist with this transition. Beginning with DERIVE 5, it became possible to save a worksheet session, including graphics, and also export the expressions and graphics to other programs (such as word processing). Today, DERIVE 6 looks very similar to DERIVE 5, with the mouse—driven toolbar, dropdown menu options and window tiling features.

DERIVE is a program which offers numeric and symbolic computations over the real numbers, complex numbers and modular arithmetic, and also graphs in 2 and 3 dimensions. In addition, it performs parametric and polar computations and plots. The advantage it holds over most graphics calculators is the ability to perform symbolic computations (the TI-89, TI-92 and Voyage 200 have this symbolic ability; their code was written in part by the DERIVE authors). The symbolic capability provides an opportunity for deeper exploration of the mathematics, yielding an improved synthesis of the numeric, symbolic and graphical approaches to mathematical learning.

Let us examine some basic features of DERIVE and how they can enhance the learning of mathematics. To begin, the plotting window allows a link between the symbolic and the graphical. As an example, suppose we want to investigate the polynomial $y = x^4 - 4.7x^3 - 2.3x^2 + 22.2x$. Factoring by hand might be a real challenge to youngsters, but DERIVE handles it easily. Using a split screen, the teacher can explore the link between the symbolic factors and the *x*-intercepts of the polynomial. Figure 1 gives a view of the four factors and their corresponding intercepts.

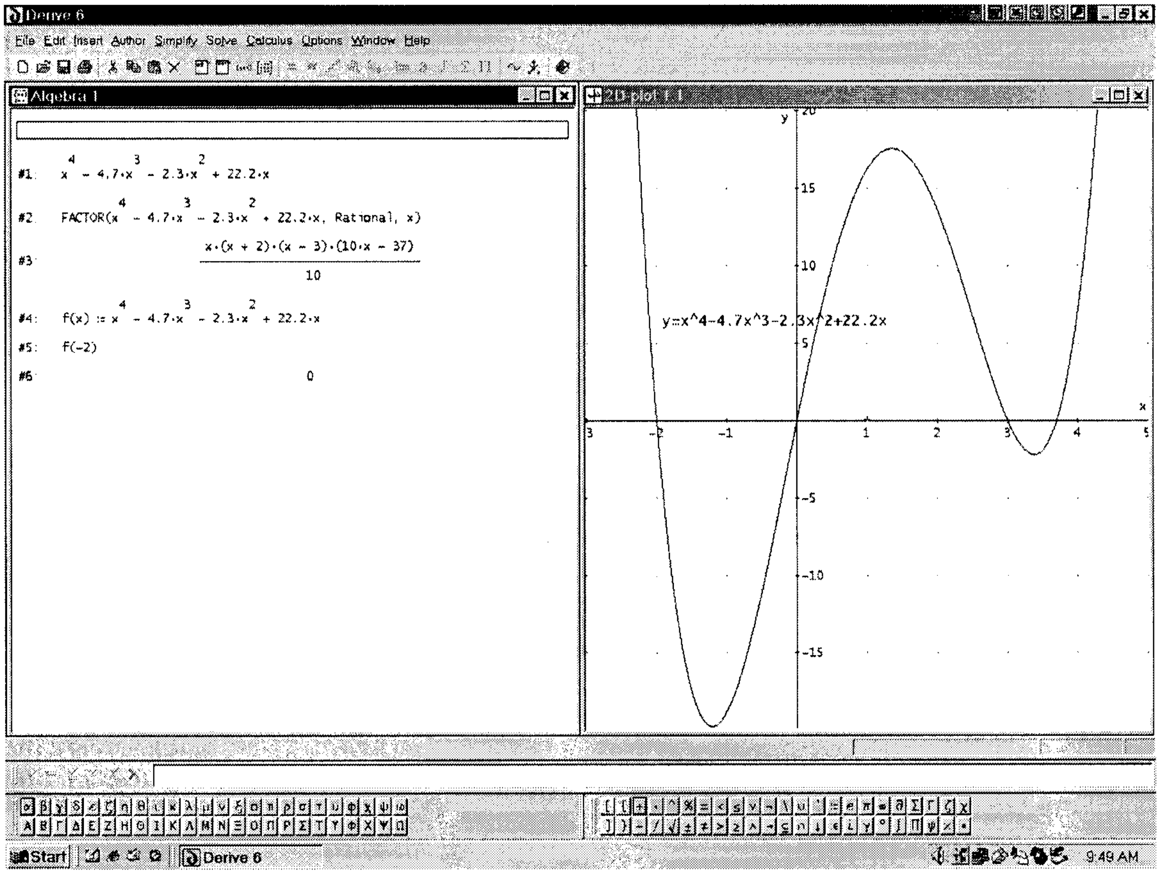

Figure 1: Factoring Compared to Polynomial Intercepts

Certainly, there are many such examples from basic algebra where a graph and algebraic content can be synthesized so that the links are more apparent. The advantage of using a CAS is that the algebraic *process* (factoring) can be eliminated while the *concept* is emphasized. Students tend to focus on process rather than the content of the theorem at hand; a CAS can help them concentrate on the general theory instead. Indeed, functional notation can be emphasized when working with a CAS such as DERIVE. Rather than having functions named "Y1" and "Y2" as in a calculator, the student can name functions "f" and "g" as s/he is accustomed to seeing them in a text.

Continuing in the mathematics curriculum, the DERIVE program is useful for investigating more complicated graphics, such as piecewise-defined functions, parametric equations and polar coordinates. Other computational features of the program include the floor and ceiling functions, modular arithmetic, logarithms with arbitrary bases, and permutations and combinations. These features can be found in many calculators and CAS platforms today. I am particularly fond of the matrix algebra capabilities, as I often teach Linear Algebra. As a transition course to higher mathematics, it is my goal to emphasize theorem and concepts over computation, but the computations are essential for both developing intuition about the theory and the practical applications. DERIVE computes symbolic matrix determinants, inverses, and all such elementary matrix algebra. It is nice to note that row reduction can be performed in exact mode; hence you can find exact eigenvectors. By asking the student to perform appropriate calculations on the computer, my assessment (tests) of student understanding can include both computation and theory in a restricted time format. Whether working in Fourier analysis, group theory, linear algebra, etc, the teacher can use DERIVE to emphasize ideas over process while still using computations to illustrate the applications of higher level undergraduate mathematics.

Let us see how the new features in DERIVE 6 make some mathematical ideas easier to teach and learn. Some features are primarily for ease in teaching: there is now a scalable font so that projected images are easy to see anywhere in the classroom. Multiline editing and customizable toolbars together with online help and an *excellent* manual for first-time users make the program very accessible to both novice teacher and novice student users.

For use in the classroom, the menu bar in the algebra window now allows for stepping through computations, rather than simplifying in one step. This is an excellent tool for breaking down a process into its components, see Figure 2. One can choose whether to display the rules being used in the steps.

#1: $\frac{d}{dx}(4.7\cdot x^3 - 2.3)$

$$\frac{d}{dx}(F(x) + y) \Rightarrow \frac{d}{dx}F(x)$$

#2: $\frac{d}{dx}\frac{47\cdot x^3}{10}$

$$\frac{d}{dx}(a\cdot F(x)) \Rightarrow a\cdot\frac{d}{dx}F(x)$$

#3: $\frac{47\cdot\frac{d}{dx}x^3}{10}$

$$\frac{d}{dx}x^n \Rightarrow n\cdot x^{n-1}$$

#4: $\frac{141\cdot x^2}{10}$

Figure 2: Stepping Through Three Derivative Rules

Some nice new features were added to graphing in DERIVE 6. There is automatic labeling of any plot you produce, color coded to the graphing object. A direct interface with DPGraph (see www.dpgraph.com) is available, and 3-D graphs can be rotated via the mouse rather than the arrow keys on the keyboard. And the addition of slider bars to the graphing windows makes parametrization come alive with animation. For example, one can explore the meaning of each parameter in the algebraic form of the parabola $y = a(x-h)^2 + k$. In the graphing window, you can insert a slider bar for each of $a, h,$ and k and see what effect they have on the graph of the parabola. See Figure 3 for one moment in the parametrization, students like seeing the animation. This new tool can be used to emphasize student exploration and discovery of mathematics rather than requiring teachers to lecture on content. The student can pose a theorem, others can ask "what if", and they can determine (with appropriate guidance) the content on their own. This slider bar feature also enhances more advanced student mathematical research.

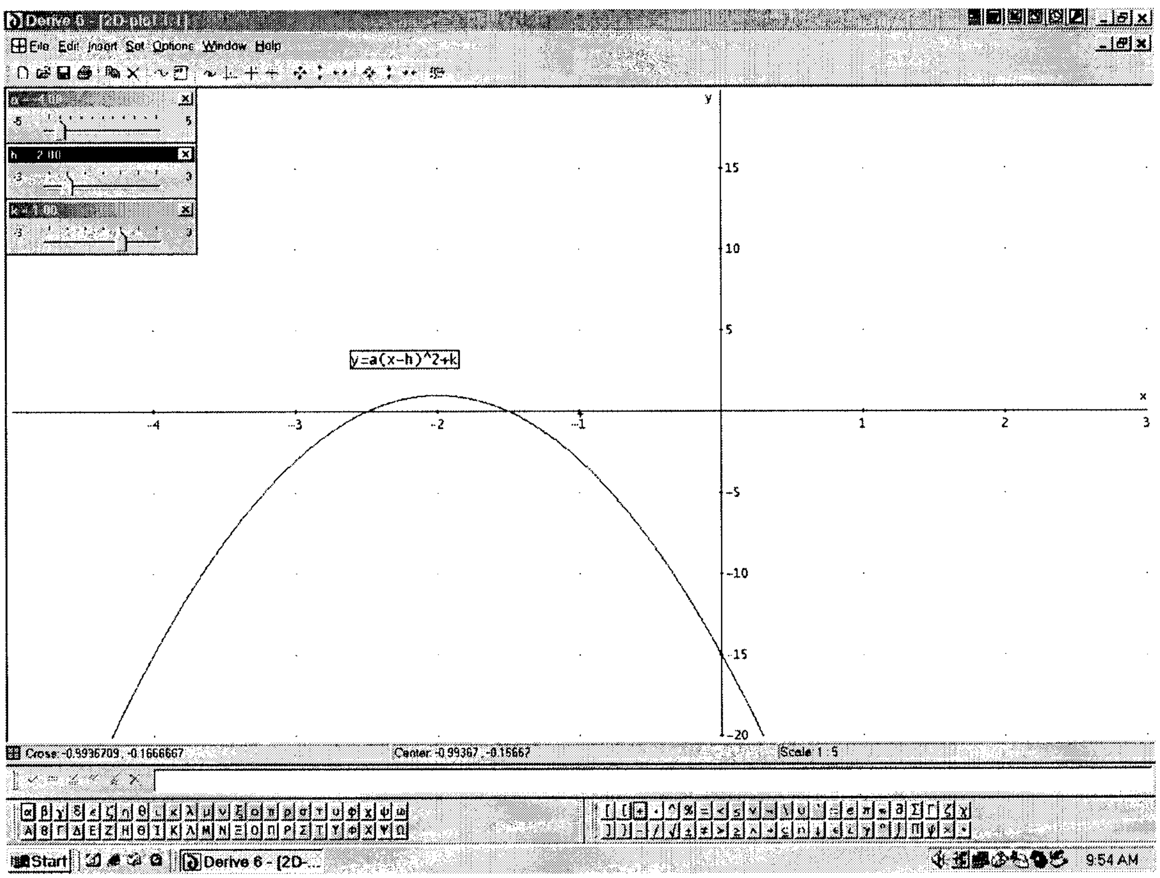

Figure 3: The parabola $y = -4(x+2)^2 + 1$

Some final new features in DERIVE 6 are important to mention. The utility files continue to be expanded. Groebner basis computations can now be performed in DERIVE, as well as solving some nonlinear systems of equations. It is now possible to import other objects into the DERIVE worksheets. A nifty demonstration of this was shown to me: import a graphics image of a photo of an everyday object into a graphing window (in the demonstration I witnessed, it was a high-heeled shoe). Then have the students model a function for this curve. The importation of text and picture objects allows one to make some very nice presentations in an algebra window that look like powerpoint but also allow one to actively continue the mathematical computations. Finally, the DERIVE 6 program can now interface with some Texas Instruments calculators with symbolic ability: the TI-89, TI-92PLUS, and Voyage 200. This gives the program greater portability, since you can work on a handheld CAS anywhere and then upload it to DERIVE (or vice versa). The following screenshot (Figure 4) from a TI-89 would upload into DERIVE 6 as Figure 5.

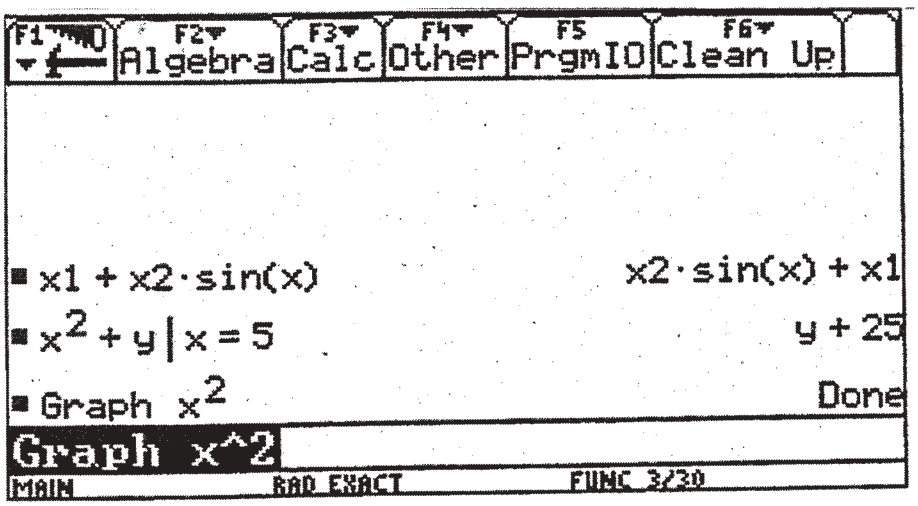

Figure 4: TI-89 Commands

```
#1:   x1 :=

#2:                                                    x1 :=

#3:   x2 :=

#4:                                                    x2 :=

#5:   x1 + x2 • SIN(x)

              2
#6:   SUBST(x  + y, x, 5)

Highlight, then press appropriate toolbar graph button:

       2
#7:   x
```

Figure 5: Exported TI-89 Commands to DERIVE