

USING *DERIVE*TM 6 IN LIBERAL ARTS/BUSINESS CALCULUS

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Introduction

For years, *Derive* has been an excellent supplement to a Liberal Arts/Business Calculus course, in part because of its ease of use compared to other computer algebra systems. Some of the new features in *Derive 6* open additional opportunities for classroom demonstrations and student learning. Among these new features are slider bars in 2D and 3D graphs, the “display steps” feature, and communication with certain TI handheld devices. In this paper, we will discuss some of the specific applications of *Derive* that we are currently using in teaching our respective calculus classes.

Slider bars tie the changing of parameters in an equation with immediate graphical results. Applications of slider bars include exploration of the effects of translations and reflections on a graph of an equation, demonstration of the convergence of secant lines to a tangent line, and exploration of graphs of functions of two variables. The “display steps” feature is most useful for students learning to apply derivative theorems, such as the product and chain rules. Our students do not use the TI handheld products (TI-89, TI-92 Plus, TI Voyage 200) that can communicate with *Derive*, so we will not be discussing this feature here.

We use *Derive* mostly to guide students to discoveries about calculus topics and to allow students to focus on theory without getting caught up in computational procedures. One of the consequences of the nonstandard notation used by *Derive* is that students are forced to recopy *Derive*'s results onto their papers; therefore students are forced to think about the meaning of mathematical notation and they gain experience in interpreting results. Additionally, students can use *Derive* to generate graphs that they can easily paste into typed homework documents.

Tangent Lines

Watching a sequence of secant lines “become” a tangent line is a powerful introduction to the derivative as a limit. Slider bars allow us to seamlessly move one of the defining points of a secant line along a curve until it is extremely close to the other defining point, thus creating a line that is essentially a tangent line.

The process described below builds a demonstration showing secant lines through $(a, f(a))$ and $(a+h, f(a+h))$, where $h > 0$. These lines will converge to the tangent line at $(a, f(a))$. A simple modification allows us to consider the limit as h approaches 0 from the left. Initially, it is important to choose a simple function such as $f(x) = 4 - (x - 2)^2$. In later examples it may be appropriate to show tangent lines at a point of inflection or on a linear function. An example showing a point of nondifferentiability can also be instructive.

Before going to class, we create a worksheet containing the following:

- A defined function $f(x)$ for which we will draw secant lines.
- The defining points of the secant line: $[a, f(a)]$ and $[a + \frac{h}{50}, f(a + \frac{h}{50})]$. When we create slider bars in the plot window, the minimum and maximum values of h must be integers. Since $h = 0$ causes our secant line to disappear, we use $h = 1$ as our minimum value, and use $h/50$ instead of just h in our defining equations.
- A defined “slope” function $m(a, h) = \frac{f(a + \frac{h}{50}) - f(a)}{\frac{h}{50}}$ that will give the slope of the secant line. The equation of the secant line: $y - f(a) = m(a, h)(x - a)$.

Since we can't save plot settings in *Derive*, we have to set up the plot window when we get to class and open the secant lines worksheet we saved previously:

- Select “large” points from the Options ... Display ... Points menu.
- Set an appropriate plot range for our function and set the aspect ratio to 1:1.
- Insert the slider bars for a and h . The maximum number of intervals for a slider bar is 50. We use a relatively small number of intervals for a . In order to have the final secant line in the sequence look essentially like the tangent line, the distance between the two defining points must be quite small. So we set the bar for h so that $1 \leq h \leq 51$ with a large number of intervals.
- Use the slider bars to set the values of a and h to be used for the initial plot. The value of a should reflect the point of tangency, and h should be as large as possible.
- Plot $f(x)$ and the two points. Highlight the equation for the secant line, so that it's ready to plot when the demonstration begins.
- Minimize the algebra window or hide it behind the plot window, unless we want to explain the algebra to our students.

When we're ready to show the demonstration to the students, we plot the secant line. Then we move the slider for h and discuss the results. If we feel the need, we reset h at its maximum value, and move the slider for a to another value.

Differentiation

We have had some success guiding students to discover derivative results such as the power and product rules using *Derive*. Students gain valuable experience in forming and testing conjectures, and our hope is that students will more easily remember results that they feel they had a hand in creating. As an example, we describe below an activity involving the power rule.

After using the limit definition of derivative to compute the derivative of x^2 by hand, we give students a list of power functions with larger positive integer exponents. For each function, we ask them to use *Derive* first to simplify the difference quotient, and then to compute the limit of the difference quotient as h approaches 0. Based on their results, we ask them to conjecture a formula for the derivative of x^n , where n is a positive integer. After they test their conjecture for several other positive values of n , we ask them to test it for some negative integer exponents. *Derive* returns expressions like $-1/x^2$ instead of $-x^{-2}$, so students may at first believe that their formula is valid only for positive exponents. Some students go so far as to formulate a new conjecture for negative exponents. This provides us a valuable opportunity to stress the importance of being able to change between different algebraic representations of the same quantity. If time permits, we ask students to test their conjectures for rational exponents as well.

We use similar activities to guide students to the discovery of the product and chain rules. Those activities require students to already know the derivatives of some standard functions, and so the activities don't involve difference quotients. Care must be taken to provide functions whose derivatives simplify in such a way that the product rule or chain rule is apparent from the form of the derivative. One reason we particularly appreciate the power rule activity is that it reinforces the students' experience with the derivative as a limit, and they see *Derive* as an expedient for performing algebraic simplification, rather than as a "black box" for computing derivatives based on secret formulas.

Integration

While we discuss the fact that a definite integral is a limit of sums, we usually compute definite integrals using sums only in the honors sections of our liberal arts calculus course. (The scientific calculus course is another matter, of course.) After students have done a few computations by hand, they can use *Derive* to perform algebraic simplification, allowing them to focus on the task of setting up the necessary sums. They can check their work by having *Derive* evaluate the definite integral to see if the result matches their sum.

We can also use *Derive* to guide students to the discovery of the first part of the Fundamental Theorem of Calculus: $\frac{d}{dx} \left(\int_a^x f(t) dt \right) = f(x)$. This is conceptually

difficult for students, so care must be taken to choose appropriate functions $f(x)$, and many students need help to correctly interpret their results.

Multivariable Functions

Students in the Liberal Arts/Business Calculus course often have difficulty visualizing graphs in three dimensions. *Derive*'s 3D plotting capabilities, along with the use of slider bars, can help students to understand the nature of graphs of functions of two variables. For instance, to demonstrate the idea of level curves, we can plot the graph of a simple function $z = f(x, y)$, along with the graph of the plane $z = k$. By attaching the variable k to a slider bar, we can create a simple but effective demonstration of the level curves for the surface. *Derive*'s ability to rotate the graph in real time assures that students are able to understand the graph, since they can see it from a variety of viewpoints. Figure 1 shows this for the graph of $z = 2 - x^2 - y^2$.

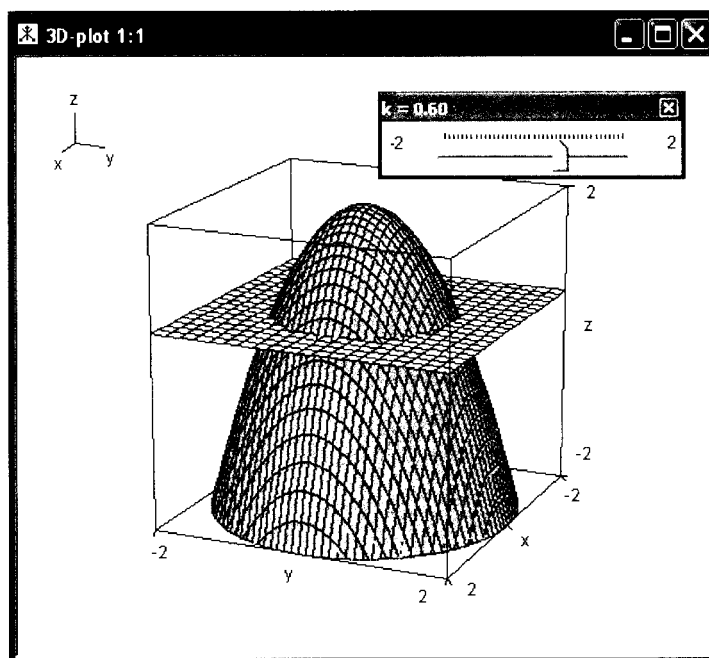


Figure 1: A slider bar being used to illustrate a level curve

Another feature of *Derive* that is especially useful in connection with functions of two variables is the ability to illustrate cross-sections of a surface, by planes of the form $x = k$ or $y = k$. Using the “trace” feature, in conjunction with *Derive*'s ability to rotate a 3D graph, we can help students to visualize the meaning of partial derivatives as slopes of tangents to cross-sections, and to understand that critical points serve to identify possible locations of relative extrema. Figure 2 shows a graph being traced. The thin white cross-section curves are easily adjusted by using the “Plot Tracing” buttons.

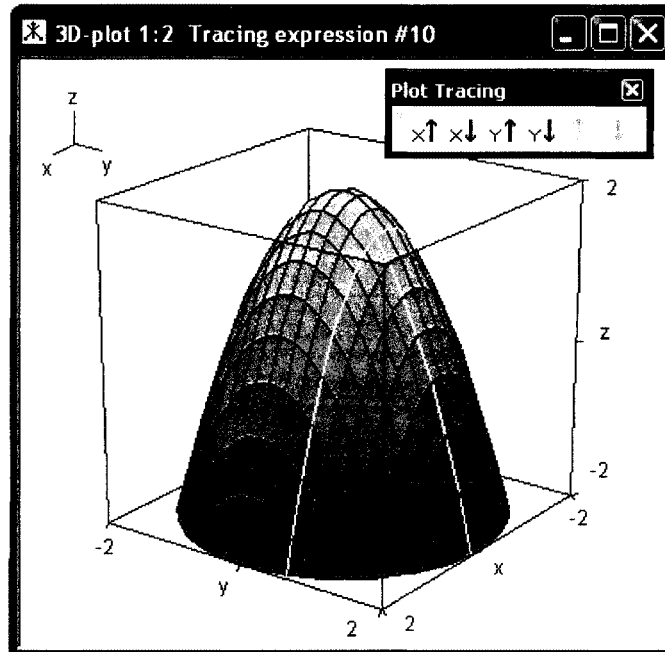


Figure 2: Tracing a 3D graph

Conclusion

Appropriate use of *Derive* can enhance students' learning experiences through both class demonstrations and discovery activities. Of course, careful planning is needed in order to time the assignments of some of the discovery exercises – i.e., if the power rule discovery is a homework assignment, it should be due before the students read about the power rule in the textbook. (We never tell the students the names of the results they are to discover, so they are unlikely to look ahead in the book for the answers.) By shifting the burden of mechanical computations to *Derive*, we free students to focus on the larger concepts of calculus.

Copies of the *Derive* activities we have used in our classes are available online at <http://www.jcu.edu/math/ICTCM2004>. A free 30-day trial copy of *Derive* is available from Texas Instruments, at <http://education.ti.com/derive>.

References

- [1] D'Ambrosia, Barbara K., and Carl R. Spitznagel. "Discovery Learning in Calculus with the TI-92." *Proceedings of the Ninth Annual ICTCM*. 1998. 96-100.
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