EXPLORING MULTIVARIABLE CALCULUS WITH DPGRAPH

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Introduction

At the Xavier University of Louisiana, the computer lab component was incorporated in multivariable calculus (MATH 2080 - Calculus III, 4 credits) in fall 2001. Over the course of years of teaching multivariable calculus, it was noted that one of major roadblocks for success in this course are students' difficulties with conceptual understanding of three-dimensional space, in particular with visualization of objects in it. To overcome this problem, a set of computer lab activities together with accompanying handbook were developed to enhance students' familiarity with objects in three-dimensional space through discovery and exploration. Materials could be integrated in courseware in conjunction with any standard textbook. The development of course materials was supported by the curriculum development minigrant from Xavier University. In 2004, the course materials are revised and the course portfolio developed. In two instances the assessment has been conducted: in fall 2001, when the course materials are introduced for the first time, and recently in spring 2004.

In the process of selecting visualization software, one may first think of some of the commercial computer algebra systems. However, it requires students to have at least basic operational knowledge of such software. Due to lack of spare time for teaching students basic on computer algebra systems, this was abandoned and the software DPGraph (Dynamic Photorealistic Graphing) was selected. It is a software package, developed by David Parker, for interactive visualization and animations of objects in two- and three-dimensional spaces, including solids, surfaces, curves, and vector fields. In addition to rectangular coordinates, it supports equations and inequalities written in polar, cylindrical, spherical coordinates, as well as parametric curves and surfaces. The software is very easy to learn and use, has a remarkable graphics capabilities, students can familiarize with it without difficulties, it provides a user-friendly environment for variety of explorations. Various built-in capabilities such as rotation, slicing three-dimensional objects, changing prospective and views, interactively varying parameters, In addition, there is an abundance of already developed materials; price is modest, with good support, and unlimited access to upgrades.

In this paper, we give a brief description of activities and course materials already developed for exploration of topics in multivariable calculus with DPGraph. In addition, we present the results of the conducted assessment and the summary of students' responses about the effectiveness of computer lab assignments and the quality of developed course materials.

Developed Course Materials

The developed course materials consist of DPGraph files used in exploration of variety of topics from multivariable calculus, and accompanying handbook. Each assignment covers material in single topic and consists of three to six independent exercises. Some exercises are designed in a way to reinforce topics already covered in the class, while some other are to introduce new concepts through explorations and discovery. Graphical, numerical, analytical, and verbal approaches are incorporated. In some exercises, students are guided to make predictions by exploring objects graphically, to describe their findings, and finally confirm them analytically. Each chapter in the handbook corresponds to a single topic. A brief review of relevant theory, including definitions, theorems, and procedures, the summary of appropriate DPGraph commands with explanations precede each assignment. In assignments students are guided step by step and the space is provided for students' answers. At the end two projects, suitable for homework or group work are added.

The following topics are covered:

- *Cross Sections and Contour Diagrams* – Students study surfaces by slicing them vertically and horizontally and analyze their contour diagrams.
- *Exploration of Surfaces* – The purpose of this assignment is to study a second-order equations (surfaces). Students make connections between surfaces (graphical representations), corresponding equations, cross sections, and contour diagrams. In addition, they analyze the effects of variations of parameters in equations, to the shape of corresponding surfaces.
- *Partial Derivatives and Tangent Planes* - Students explore tangent planes of surfaces of the form $z = f(x, y)$ and $g(x, y, z) = 0$.
- *Local and Global Extrema* – Students find critical points on a surface by exploring it together with a family of horizontal planes. They classify critical points as local maxima, minima, or saddle points, and localize them on contour diagrams.
- *Double Integrals with Applications* – Exercises explore change of order of integration in double integrals, change of variables, integrals in polar coordinates, and applications to finding volumes of solids and areas of plane regions.
- *Triple Integrals with Applications* – Triple integrals in rectangular, cylindrical and spherical coordinates are explored. One of exercises deals with the volume of an "ice cream cone" bounded from above by a paraboloid and from below by a cone.
- *Parametric Curves* – Exercises include an animation of a circular motion of a particle involving two parameters. Students explore and confirm analytically the effects of varying parameters to the motion of a particle. In another exercise students make a design of a water park ride in the shape of helicoidal tube.
- *Vector Fields* – One of the exercises deals with exploration of the effects of change of parameters *a, b, c, d* to two-dimensional linear vector filed $\vec{F} = (ax + by)\vec{i} + (cx + dy)\vec{j}$.
- *Line Integrals* – The effects of change of path to the value of line integrals of some vector fields was investigated in one of exercises.

In addition to above activities two projects aimed as homework assignments or group work, are also designed. The first one deals with exploration of well-known Viviani's solid defined by $x^2 + y^2 + z^2 \le a^2$, $x^2 + y^2 \ge ax$, and $x^2 + y^2 \ge -ax$, where $a > 0$. In the other the vector field $\vec{F} = a\vec{r} / \|r\|^d$ where a, and d are real numbers is investigated.

Sample exercises

In this section, we present few sample exercises.

Exercise 1. Open file lab02-e02.dpg in DPGraph and examine the surface described with the equation $ax^2 + by^2 + z^2 = c$, where a, b, and c are real numbers and complete the table

a	b	c	Equation	Name	Cross section		Contours
					x constant	y constant	
1	1	1					
1	1	0					
⋮							

Exercise 2. Open file lab02-e04.dpg in DPGraph that represents a picture of a "cherry in the empty glass". Identify all surfaces that appear on the picture.

Figure 1. The picture of cherry in the empty glass" in two different views.

Exercise 3. Open file lab03-e02.dpg. The graph represents the surface $z = 1 - x^2 - y^2$ and the plane $z = 1 + a(x+1) + b(y+1)$ that both pass through the common point (0,0,1). Answer the following questions.

1. Vary parameters a and b so that the plane is tangent to the given surface at the point (-1,-1,-1). Vary parameters a and b. Use arrow keys to rotate the picture in order to get a good view. Write the equation of the tangent plane.
2. Now find analytically z_x, z_y and evaluate them at (-1,-1) and write the equation of the tangent plane.
3. Compare the result obtained analytically with the one obtained graphically.

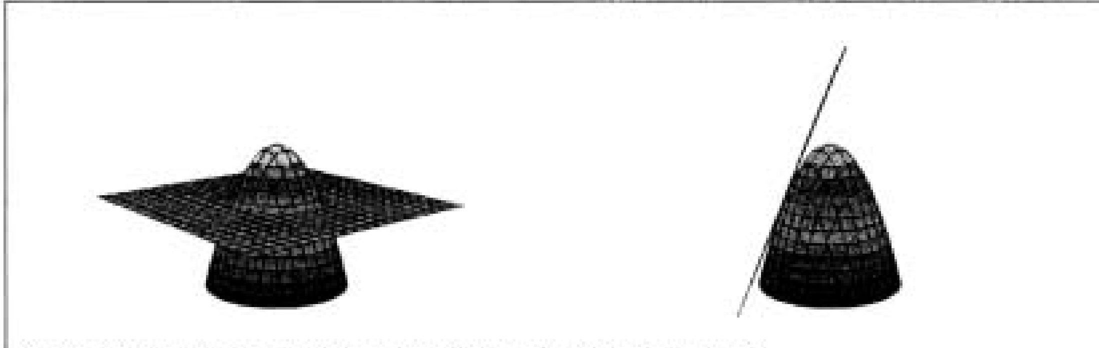

Figure 2. The surface and the plane for a = b = 0 and a = b = 2.

Exercise 4. Open file lab04-e01.dpg. The picture represents the surface $z = \frac{1}{3}(x^3 - 3x + y^3 - 3y^2)$ and horizontal plane $z = a$. Explore the surface by changing the values of parameter a. Locate critical points, classify them as local maxima, local minima, or saddle points.

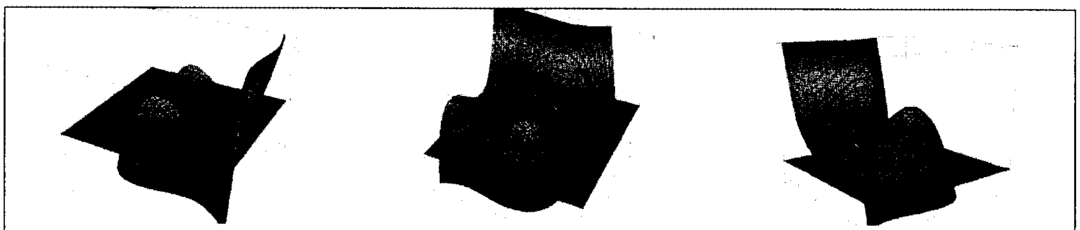

Figure 3. Surface and the horizontal plane for three different values of the parameter a.

Assessment

In assessing the effectiveness of computer lab components two main tools were used:

1. *Daily journals* - At the end of each lab students will answer few short questions about the effectiveness of that particular class and goals achieved.
2. *Comprehensive survey* - At the end of the semester a detailed survey (more than 50 questions) is conducted to address overall effectiveness of the project. The survey was divided into four main parts targeting various parts of the projects: (i) handbook; (ii) visualization in two- and three-dimensional spaces; (iii) class structure; (iv) possible improvements.

The assessment was done twice, in fall 2001 and spring 2004. In both cases, there were no significant differences among students' responses. Here are some findings summarized from spring 2004.

- Students were very satisfied with the content and structure of handbook. The handbook helped majority of students to better understand key concepts in the course. They do not recommend elimination of any part. Review was very beneficial for students both in the lab and for future references.
- Assignments were understandable, clearly written, with the description of commands to be used for their completion. They helped students in understanding of applications

of Calculus III. Assignments were somewhat challenging for half of the student body, and not always related to the appropriate topic. Most assignments were related to entire chapter and in some cases (because lab met only once a week) students were not previously exposed to the material.

- Visualization using DPGraph software significantly helped students to improve their understanding relationships between objects (curves, surfaces, solids) in three-dimensional space. Examples were interesting and illustrative. In particular, students emphasized the quality of graphical representations, animations, and prepared examples.
- Students were satisfied with the work in the computer lab. They agreed that the most important topics are covered. Labs helped them to significantly improve their performance in the course. The weight assigned to computer lab component in calculating final grade was appropriate. Grading policy was fair. They found that the time allocated for labs was not appropriate – labs were too long.
- As possible improvements students suggested better correlation between labs and material covered, incorporating labs into lectures, and more projects (group and individual). The need for the group work was also stressed. They also suggested that the labs should include mostly short examples, but should contain some complex problems.

Conclusion

The computer lab component involving explorations in multivariable calculus using DPGraph was success at the Xavier University of Louisiana. This is still on-going project and in need of developing additional activities targeting topics not currently covered.

References

1. V. L. Kocic, *Exploring Multivariable Calculus with DPGraph*, Handbook, Xavier University of Louisiana, 2004.
2. D. Parker, *DPGraph*, www.dpgraph.com