MODELING PROJECTS FOR STUDENT INVESTIGATION

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The following projects are intended for the typical 1^{st} year integral calculus student studying Applications of Integration - a standard textbook chapter in such a course. The projects necessitate using a computer algebra system to assist in the modeling of "real data" in the following three situations: Center of Mass, Average Value of a Function, and Volumes of Revolution. Then, using theory from the course, students are asked to find a "solution" to their project problem.

As opposed to the typical 1^{st} year course in differential calculus, the integral calculus course has a plethora of *real*ly nice applications. Many of these are applications of integration and most of the popular calculus textbooks devote a chapter or two to this topic. The following three projects fall under this category. They get students to physically address the questions at hand. They also involve competition and food - two topics that are dear to the hearts of most college students.

Each of these projects requires approximately 30 to 60 minutes to complete. They are self-contained projects that require the students to have access to a computer algebra system (CAS). For my course and this paper, Mathematica is the CAS of choice. The CAS is used to "create" a function from data that the students collect. Typically, two or three students work together as a team on each project.

Fruit (and Vegetable) Volumes of Revolution - This project is appropriate for students studying volumes of revolution. Each team of students is given a symmetrical piece of fruit. Although spherical pieces (oranges, grapefruits, etc.) can be used, they are trivial for this project. Many other symmetric fruits usually do not work well either (as will become obvious later). These include apples or anything with a hard pit. My fruit of choice is the pear. It is available year-round, has no pit, exhibits symmetry, and is delicious. (Vegetables, such as eggplant or zucchini, also work well.)

The students are asked: What is the volume of your pear?

Each team is supplied with a pear that has been cut in half lengthwise and a piece of graph paper. They are given instructions on how to trace their pear to get a picture of their "pear function." For example, the students are told to align the main axis of their pear with the x-coordinate axis on their graph paper and then trace one half of the pear onto the paper. See Figure 1. Using the grid on their graph paper and their tracing, the students identify the coordinates of points on their pear curve. These points are the input for Mathematica's Fit command that produces an equation, a "fit" function, for the pear curve. Each pear curve will yield a function - or it will after a little "massaging" of the data. Other fruit, such as apples, may have decidedly non-function tracings. Mathematica uses a basis of functions, chosen by the student, to model the data. Typically, this may include power, trigonometric, exponential, and logarithmic functions. For example, in Mathematica,

$$\text{Fit}[\textit{data}, \textit{funs}, \textit{vars}]$$

finds a least-squares fit (a "fit" function) to a list of *data* as a linear combination of the functions *funs* of variables *vars*. Students then apply the formulas learned in class to solve the appropriate definite integral of the fit function to approximate their pear's volume. Depending on the kinds of functions used to form the fit function (and the goals of the instructor for this project), students can either work on their definite integral by hand or again use Mathematica.

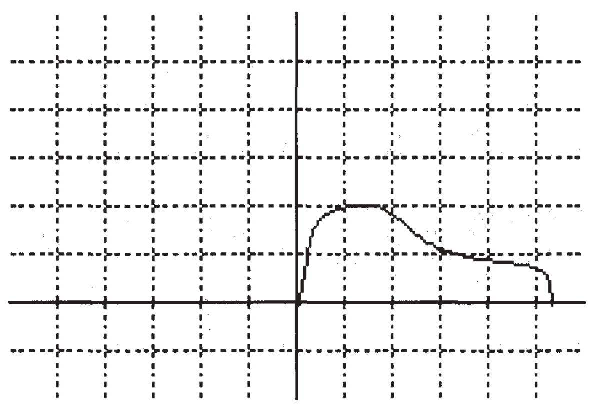

Figure 1: A typical pear function tracing.

To provide some sense of checks and balances for their work on this project, a calibrated beaker containing water is also provided. Teams submerge their pear into the water to determine its volume. They are asked, in their project write-up, to discuss which method, volume of revolution or submerging in water, they believe is more accurate in determining volume. It is surprising to me to see how many students say that submerging their pear is less accurate than their use of the volume of revolution method. It is probably a good sign that the calculus has such high credibility with them.

This project has been quite successful with my students. They feel that finding the volume of a pear has some merit. And, they are not surprised to see pears show up in one of their projects. I also use them in class when introducing volumes of revolution. I slice a pear more and more thinly to show the students how the pear's curved surface can be better and better approximated by a straight line when the pieces of pear get thinner and thinner. It should also be noted that this project can be used to foreshadow the topics of Taylor and Fourier series.

Average Temperature Value of a Function - In this project, each team of students is given a handout with a temperature graph for some 24-hour period. See Figure 2. Barometric pressure, wind speed, and relative humidity graphs also work nicely. These types of graphs are readily available on the Internet. I find that either a local temperature graph or one from some distant country create the most interest.[1] The teams of students are asked to find the average temperature for their day in question. (Before starting the project, I sometimes motivate this question with a discussion on finding averages of discrete data (e.g., bowling scores).)

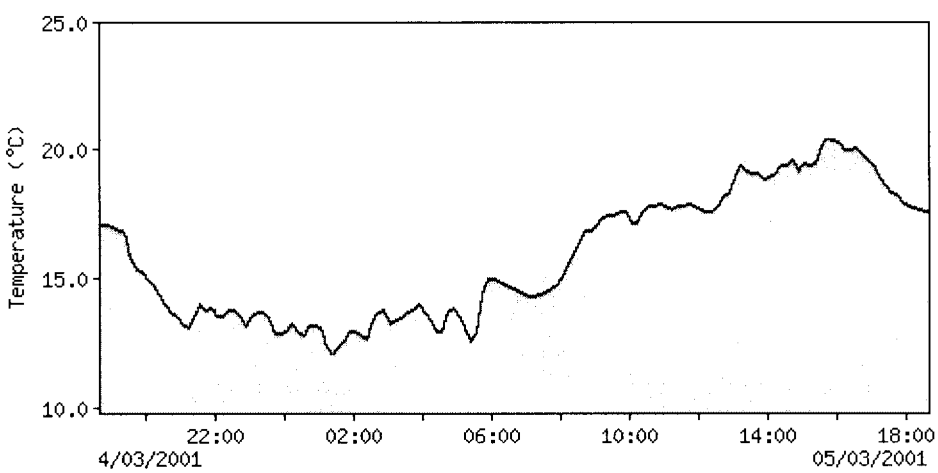

Figure 2: A sample temperature graph.

Once again, the students are prodded into putting their temperature graph on a grid and identifying a number of data points. They then, with the help of Mathematica, use known functions to produce a "fit" function. Students then apply the average value formula learned in class to solve the appropriate definite integral to approximate their day's average temperature. Again, depending on the kinds of functions chosen to form the fit function (and the goals of the instructor for this project), students can either work on their definite integral by hand or use Mathematica.

Most often, the actual average temperature for the temperature graph will also be available and this provides the goal for the students' work. Student grades can be based on the absolute difference between the actual average temperature value and their calculated value. E.g., within .5° earns an A, between .5° and 1° earns an B,

[1] I often use meteorological graphs from the National Technical University of Athens's website (http://www.meteo.ntua.gr/e/charts/). Their meteorological station is located on the west foot of Mount Hymettus.

between 1° and 1.5° earns a C, etc. Student interest in this project is quite refreshing. However, when the project is returned, I oftentimes tell the class that average temperature is usually determined from sampling a great number of points, equally spaced over the course of a 24-hour period and merely averaged. This is actually their intuition at the start of the project especially when motivated by the bowling scores example. They enjoy the project nonetheless.

Center of State Mass - Each team of students is presented with a piece of paper on which is drawn the outline of a U.S. state. See Figure 3. Their task is to find the center of mass of their state, assuming it is actually a lamina with uniform density. One benefit of this approach is that the students cannot merely put their finger under their state and, through trial and error, find the balance point, i.e., center of mass. They only have a picture of their state drawn on a piece of paper and not, say, a cardboard cut-out of their state.

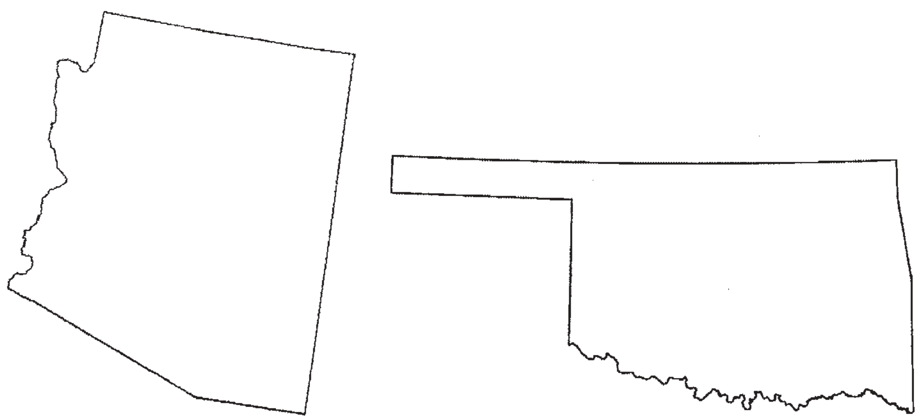

Figure 3: The outlines of Arizona and Oklahoma.

As with the previous two projects, teams start by superimposing a coordinate system on their state and finding points with which to use in Mathematica's Fit command. However, many states may need some manipulation (rotation and/or cut into smaller pieces) to facilitate the modeling process, i.e., to be represented by a function(s). For example, teams may want to begin by rotating Arizona a little under 90° clockwise or cutting Oklahoma into two pieces with one piece being the panhandle. In addition, as in the Volume of Revolution project, the outline of a state (or each piece of a state) may need an extra adjustment so that the data points are representing a function(s). If the adjustment will not be minor, then more manipulation of the kind mentioned above would be in order. Finally, the data points are used as input to Mathematica's Fit command. Using the command's output, i.e., a "fit" function, the students can then apply the theory developed in class to find their state's center of mass. This point is marked on their paper.

To check their work, I sometimes have a second piece of paper on which is the outline of their U.S. state as well as its actual center of mass. (The "center of mass" or geographic center of each state is available on the Internet.) We superimpose the two state outlines to see how close the students' calculated center of mass point comes to the actual point. Concentric circles around the actual center of mass can be drawn to gauge a student's accuracy, i.e., to determine their grade. Othertimes, I make a cardboard cut-out of their state and the student teams mark their calculated center of mass point on the cardboard. Then the cardboard is placed on a nail driven into a piece of wood with the calculated center of mass point on the nail head. The hope is that it balances.

Each of these three projects uses the same basic idea of modeling data with functions. This is an important topic that is often overlooked in the calculus sequence, which is surprising given the emphasis in the last few years of the importance of applications to motivate student learning. The techniques involved with modeling data with functions are ones that are readily accepted by students and can be formally addressed in the calculus sequence or in another course.

It is interesting to note that many student teams, after Mathematica produces a "fit" function from their set of data points, will use Mathematica to plot the fit function to see if it matches their pear tracing or temperature graph or state outline. They find this a good verifier as to whether or not what they are doing is correct. (They need to be careful with things like axis scaling and aspect ratio when comparing their curve with Mathematica's Plot output.) If their fit function is not accurate, the following items should be considered: (1) the number of data points chosen and (2) the basis of functions (both the type and number) chosen for Mathematica's Fit command.

All in all, these projects are very well received, all three have "correct" answers, and the students enjoy seeing theory predict reality.