

A TECHNOLOGY-ENHANCED FRACTAL/CHAOS COURSE

Taeil Yi
University of Texas at Brownsville
80 Fort Brown
Brownsville, TX 78520
tyi@utb.edu

Abstract

Easy construction of fractal figures is the most valuable procedure in learning Fractal geometry and Chaos theory. Calculators are not sufficient for such activities. Any powerful computer algebra system is not matching well with these kinds of activities, and students need to spend a lot of time to learn the program language and/or how to use the program itself. We introduce several free fractal constructing programs and benefits by using these programs in a fractal geometry course.

Introduction

The goal of the course is providing hand-on activities using free downloadable fractal programs as much as exploring the mathematical ideas in fractals geometry and chaos theory. **Winfeed** ([W]) and **Brazil Design** ([BD]) are the programs that we used mainly for the course. We also used **Fractree** ([F]) and **L-system** ([L]) to compare the programs.

The followings are the contents of the course and the used program(s).

Self similarity	W-IFS,B
Box self similarity	W-Dimension
Sierpinski triangle, carpet, and tetrahedron with color patterns in pascal's triangle	W-IFS,B
Chaos game in Sierpinski triangle and Sierpinski triangle addresses (for random)	W-IFS-complex,
Similarity among Trees, the Sierpinski triangle, the Cantor set, and the chaos game	W,B
Complexity in Geometric construction (Koch curve, Peano curve, several Fractal curve, zigzag, etc)	W-string rewriting, B
Iteration in staircase & spiral	W-web diagram
Graphical composition & iteration	W-web diagram
Fixed points as attracters & repellers Solving Equations and finding pre-images	W-web diagram, B
Box dimension for any geometric shapes	W-Dimension W-string rewriting- box count
Attracters, chaos, and expansions & compression for $F(x)$	W-web diagram

$= ax(1-x)$	
Sensitivity	W-web diagram
Iteration of $f(x) = x^2 + c$ and $f(x) = px^2+qx+r$	W-web diagram
Graphical composition machine	
Mixing, Periodicity and Sensitivity of the saw-tooth function and tent Function	W-web diagram
Transformation between tent and parabola	W-web diagram
Long term behavior & time series	W-web diagram
The Feigenbaum plot	W-Bifurcation
Prisoner set & invariant intervals	W-web diagram
Invariant intervals and Cantor set	
Critical points of $f(x) = ax(1-x)$	W-web diagram
Maximal invariant interval	W-web diagram
Rate of escape of $f(x)=x^2+c$	W-web diagram-table orbit
Orbits of $y=z^2+c$	W-IFS-complex
Mandelbrot set	W-Mandelbrot
Julia set	W-Julia

Table 1 - Contents and programs

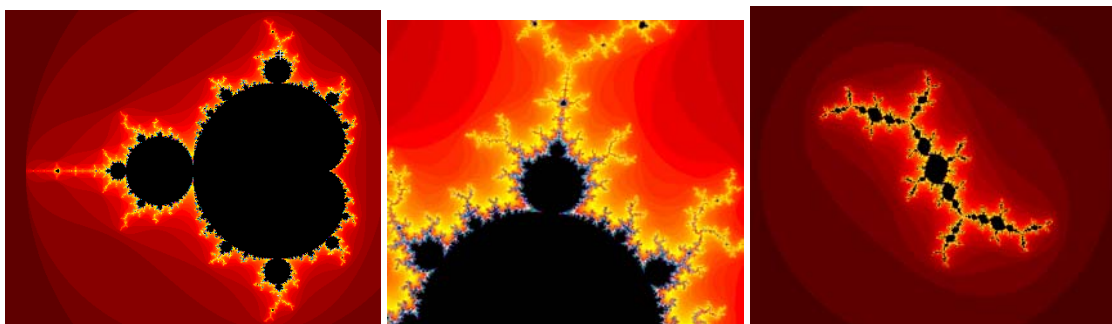


Figure 1 – Mandelbrot set and Julia set *done by W-Mandelbrot*

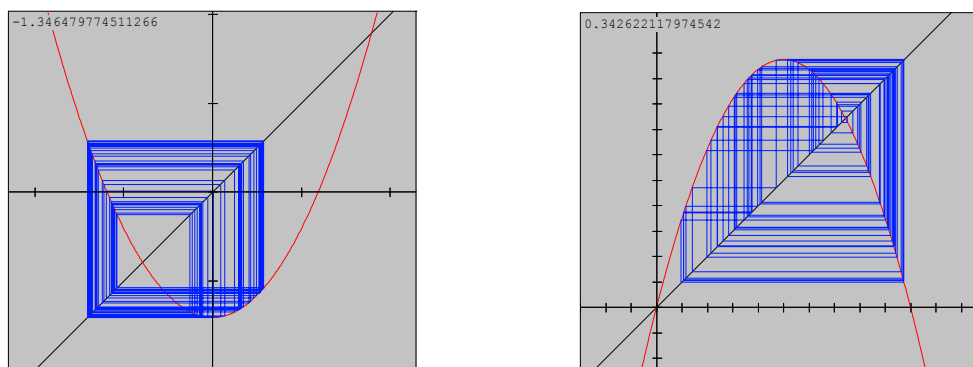


Figure 2 – Web Diagram for $y=x^2-1.415$ and $y = 1.415x(1-x)$ *done by W-Web diagram*

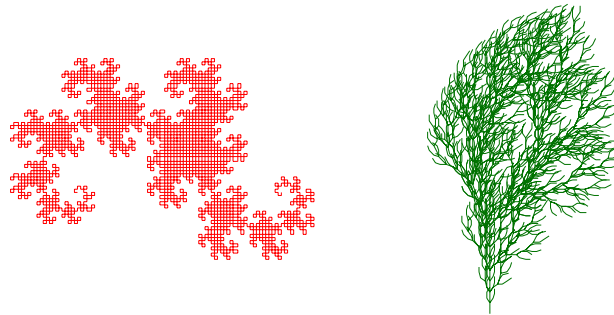


Figure 3 – String-Rewriting for dragon and bush *done by W-String Rewriting*

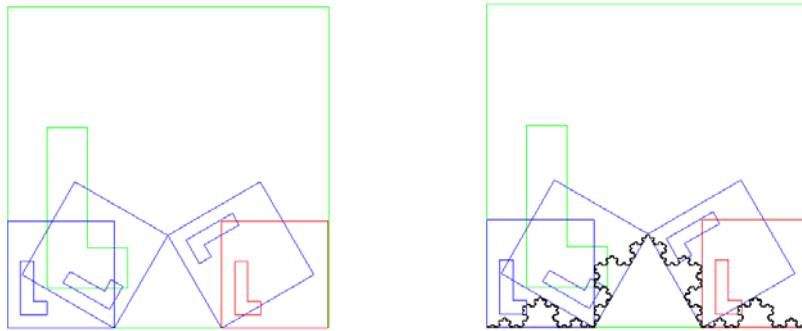


Figure 4 – Transformational Design for Kock's Snow Flake *done by Brazil Design*

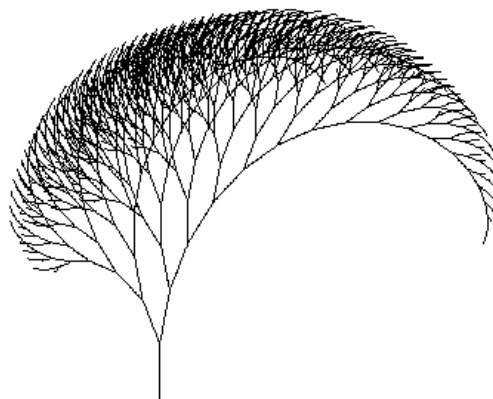


Figure 5 – String-Rewriting for stylish tree *done by L-system*

Problems in Fractal teaching/learning

In the procedure of learning fractal geometry, observation of the construction of fractal images is the most important part as much as understanding mathematical ideas behind them. Students will understand mathematical concepts better if they could see the construction procedure of the fractal figures step by step. Even though almost all of fractal books come with a lot of figures, it is not enough yet, because students cannot see the actual construction and/or the movement. It is very difficult for students to see the relation between the mathematical concepts and the figures in the book.

Some books suggest using some fractal softwares. For example, Barnsley's book ([B]) can be supplemented by his Desktop Fractal Design System which is not free. Some other books give computer algorithms for fractal shape construction. The main problem is that it is difficult to manipulate the algorithm to construct different fractal shapes other than the given one if students are not familiar with the program language. If the students cannot see any changed fractal figures from the original one, then there is no difference between the computer generated fractal figures and the one in the book. If the instructor wants the students who are not familiar with the programming language to learn the programming language, then students may need more time to learn programming language other than the mathematical concept behind the fractal figures. Even though they understand the algorithm and the programming language, they still need to spend a lot of time for the programming to construct fractal images. Moreover, some computer algebra systems are expensive, and calculators are not powerful enough for these kinds of activities.

Role of fractal programs in the course

Winfeed ([W]) is one of free fractal exploration programs which anyone could download from the internet. It allows the user to explore functional iteration, including Mandelbrot and Julia sets, ferns and the Sierpinski shapes, web and bifurcation diagrams, string rewriting, and more. **Brazil Design ([BD])** allows the user to explore transformational iteration using geometric shapes instead of functions. **Fractree ([F])** is based on a string rewriting system which generates fractal curves and fractal trees. **L-system ([L])** reads a l-system description in and processes it into a 3d form which can then be output into DXF format.

These programs provide many features including:

- (1) Easy construction of new fractal shapes,
- (2) Final form and/or step-by-step changes of fractal images including their numerical information,
- (3) Freedom of the choice of the function, the number of iterations, orders, and seed values,
- (4) Showing Orbits and fixed points,
- (5) Providing the box dimension of any geometric images,
- (6) Easy interaction between the Mandelbrot set and the Julia set including period,

- overlay critical points, image and pre-image, and searching for finite orbits,
- (7) Zooming in and out for any constructed fractal images,
 - (8) Converting fractal figures to a PicTeX file.

Conclusion

Winfeed ([W]) and **Brazil Design** ([BD]) are very easy tools to construct and/or manipulate any fractal shapes. These programs could show not only the fractal shapes itself, but also the step by step construction of the shape. Thus the students could confirm the mathematical concept easily through the activities.

By using these fractal programs students could build fractal images almost right away without using any computer programming. So the students could focus on the mathematical concepts, not the programming. The students in the course even built their own fractal shapes right away after learned how to operate the program once which took less than an hour. They could compare the shapes they made each other. It gave them better understanding of the concept.

It is easy to distribute these fractal programs to anyone who wants to use them, since they are free and downloadable from the internet. For example, the students in the course taught the concept of fractal geometry including the construction to their own junior-high and high school students after download the program to their class computers.

The benefits of using these fractal programs in the course are the followings:

- (1) These programs are designed especially for fractal geometry, so they are easy to learn and use. No one needs extra experience of any computer programming.
- (2) Since students can change the equations and conditions within the program easily, they could make their own fractal images in no time. It gives the students better understanding about mathematical ideas of fractal geometry, because they can see the 'motions' and 'changes' of fractal shapes.
- (3) One could do the same activity for any level of junior-high, high school, and college students after downloading the programs from the internet, because these programs are free to use.
- (4) The students are well prepared for more advanced courses in fractal geometry and chaos theory, because they understand the mathematical concepts better by using the fractal programs.

Reference

- [B] Bransely, M., Fractals Everywhere, 2nd ed., Academic Press Professional, 1993
- [BD] "Brazil Design" - computer program- (David Chardonnet, 12, rue des Peupliers, 78200 Magnanville, France)
- [D] Devaney, R., A First Course in Chaotic Dynamical Systems, Addison Wesley, 1992

- [E] Edgar, G., Measure, Topology and Fractal Geometry (UTM series), Springer-Verlag, 1990
- [F] “Fractree” - computer program - (Michael Schernau, P.O.Box 5511, Irvine, CA 92716)
- [L] “L-System” - computer program - (Timothy C. Perz, <http://www.thinkpiece.com>)
- [P1] Peitgen, H.-O., Jurgens, H., Saupe, D., Chaos and Fractals, Springer-Verlag, 1992
- [P2] Peitgen, H.-O., Jurgens, H., Saupe, D., Fractals for the Classroom, Springer-Verlag, 1992
- [P3] Peitgen, H.-O., Jurgens, H., Saupe, D., Fractals for the Classroom-Strategic Activities, Springer-Verlag, 1992
- [P4] Peitgen, H.-O., Saupe, D., The Science of Fractal Images, Springer-Verlag, 1992
- [W] “Winfeed” - computer program - (<http://math.exeter.edu/rparris/winfeed.html>)