Simplex: Artificial Variables with TI-89 or Voyage 200

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Big M Method. Consider the following example.

$$\begin{aligned}
\text{Minimize } Z &= \tfrac{2}{5}x_1 + \tfrac{1}{2}x_2 \\
\text{Subject to } \tfrac{3}{10}x_1 + \tfrac{1}{10}x_2 &\leq \tfrac{27}{10} \\
\tfrac{1}{2}x_1 + \tfrac{1}{2}x_2 &= 6 \\
\tfrac{3}{5}x_1 + \tfrac{2}{5}x_2 &\geq 6 \\
x_1, x_2, x_3 &\geq 0
\end{aligned}$$

The minimization problem is transformed into a maximization form by the expression:

$$\text{Maximize } -Z = -\tfrac{2}{5}x_1 - \tfrac{1}{2}x_2$$

To transform the last constraint into an equation, we have to subtract a **surplus** variable from the left hand side,

$$\tfrac{3}{5}x_1 + \tfrac{2}{5}x_2 - x_3 = 6$$

The first constraint need a **slack** variable to become equation,

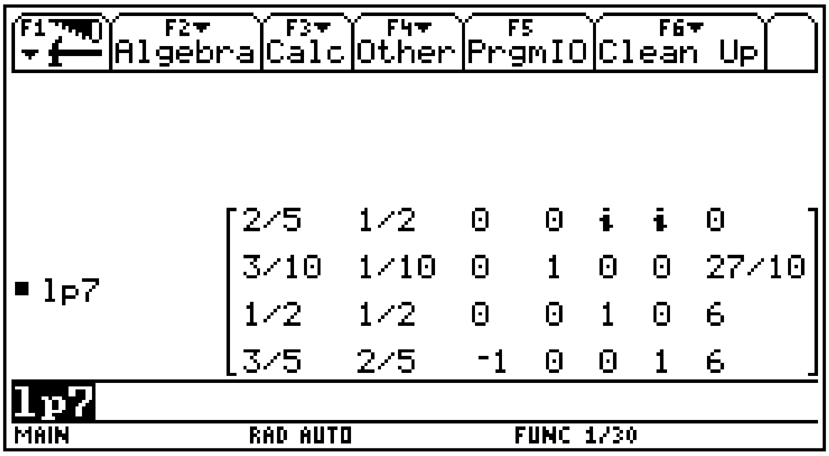
$$\tfrac{3}{10}x_1 + \tfrac{1}{10}x_2 + x_4 = \tfrac{27}{10}$$

Now all the constraints are equations but do not provide a initial basic feasible solution, hence the need to add **artificial** variables to the second and third constraints, obtaining the following augmented system.

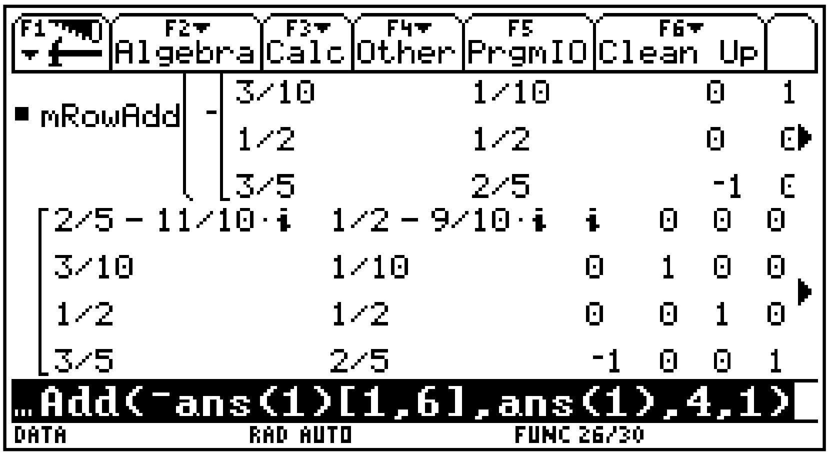
$$\begin{aligned}
-Z + \tfrac{2}{5}x_1 + \tfrac{1}{2}x_2 \qquad\qquad\qquad + M\bar{x}_5 + M\bar{x}_6 &= 0 \\
\tfrac{3}{10}x_1 + \tfrac{1}{10}x_2 \qquad + x_4 \qquad\qquad\qquad\qquad &= \tfrac{27}{10} \\
\tfrac{1}{2}x_1 + \tfrac{1}{2}x_2 \qquad\qquad\qquad + \bar{x}_5 \qquad\qquad &= 6 \\
\tfrac{3}{5}x_1 + \tfrac{2}{5}x_2 - x_3 \qquad\qquad\qquad\qquad + \bar{x}_6 &= 6
\end{aligned}$$

Notice that in the objective function artificial variables are penalized with big coefficients M, so that they will never become basic variables.

On TI-89 or Voyage 200 we will implement this **Big M** coefficient by using the imaginary unit. These calculators provide the following elementary operations that we use in this type of computations:
mRow(expr, mat, index) that multiplies the row indicated in **index** of the matrix **mat** by the expression in **expr**.
mRowAdd(expr, mat, index1, index2) that adds to row **index2** the row **index1** multiplied by **expr**.
We put the matrix in the calculator to get:



However this is not a standard simplex tableau, since the coefficients of the basic solution on the objective function have to be zero. To fix this problem we use elementary operations and get.

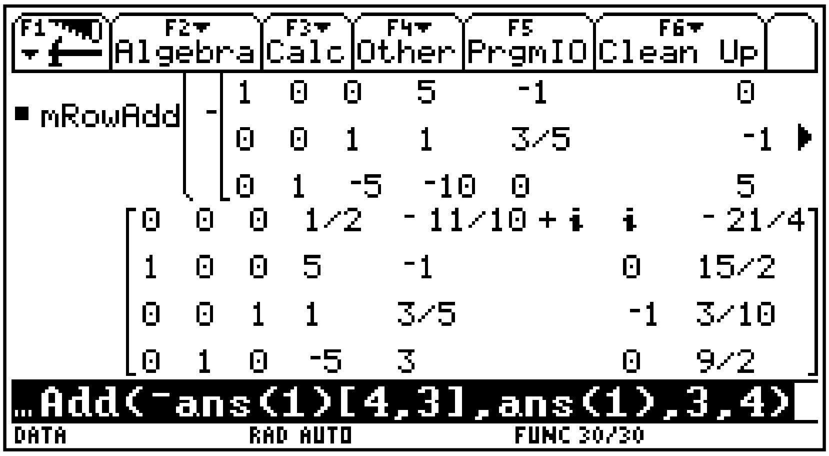


Having the standard tableau we proceed with the simplex. Assuming that the pivot is in the i^{th} row and the j^{th} column, the calculator instructions will look like

$$mRow(1/ans(1)[i,j], ans(1), i)$$

$$mRowAdd(-ans(1)[k,j], ans(1), i, k),\ k \neq i$$

After three iterations we obtain the following screen



The solution is
$-Z = -\frac{21}{4}$, $x_1 = \frac{15}{2}$, $x_3 = \frac{3}{10}$, $x_2 = \frac{9}{2}$

Two Phase Method. We run the simplex twice, the first time with

$$\text{Minimize } Z_1 = \bar{x}_5 + \bar{x}_6$$

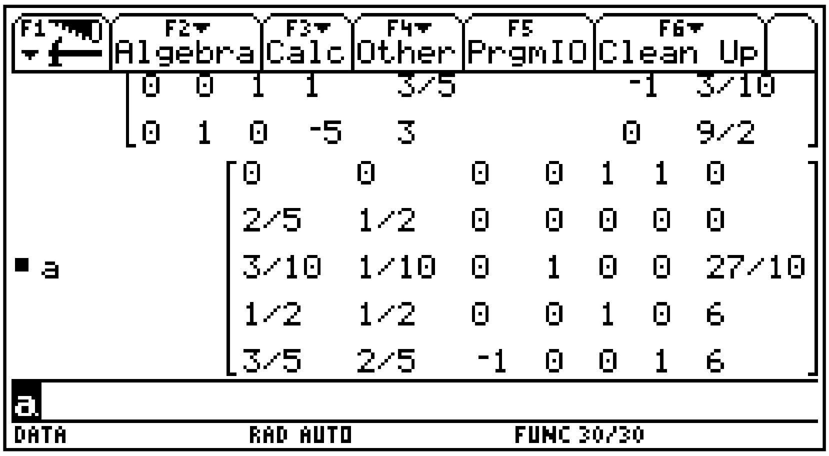
until both arbitrary variables become non-basic, and the second time with:

$$\text{Minimize } Z_2 = \tfrac{2}{5}x_1 + \tfrac{1}{2}x_2$$

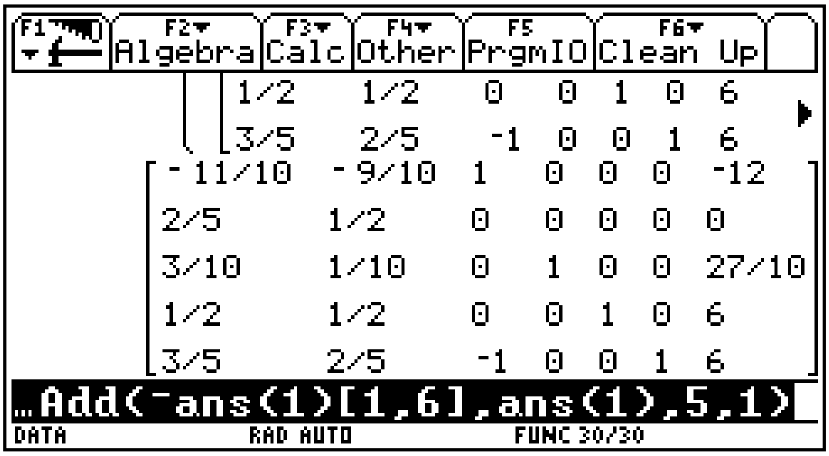
The calculator implementation will take both phases simultaneously. Changing to maximization we have:

$$-Z_1 + \bar{x}_5 + \bar{x}_6 = 0$$
$$-Z_2 + \tfrac{2}{5}x_1 + \tfrac{1}{2}x_2 = 0$$

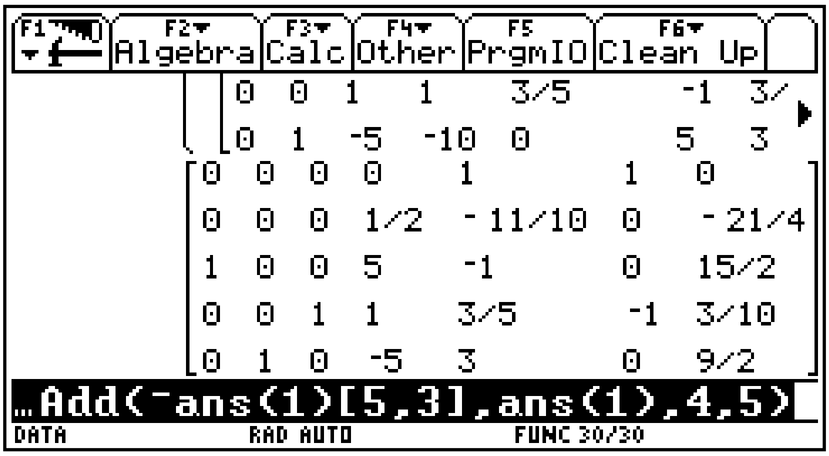
The matrix, in the calculator will look like this one:



Again this is not a standard tableau, after two elementary operations we get:



Three iterations latter we get the following screen



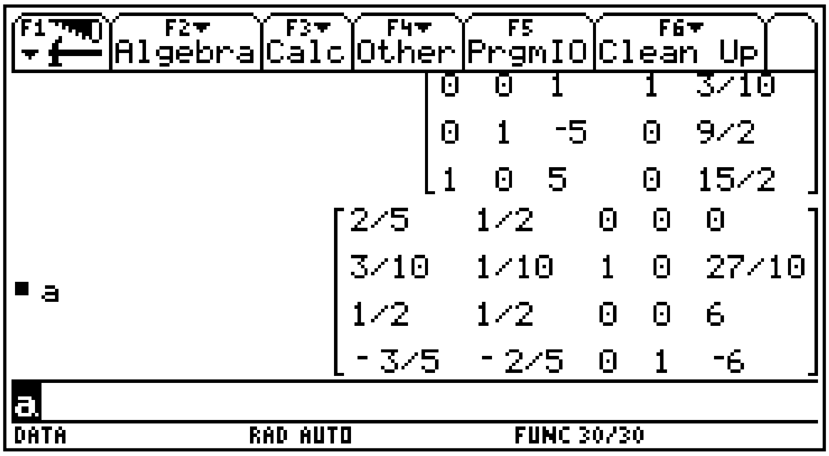
This corresponds to an optimal tableau, and the artificial variables are no longer basic variables. Ignoring the first row and the columns corresponding to artificial variables we see that the resulting tableau is also optimal, and the solution is as before.

$$-Z = -\frac{21}{4},\ x_1 = \frac{15}{2},\ x_3 = \frac{3}{10},\ x_2 = \frac{9}{2}$$

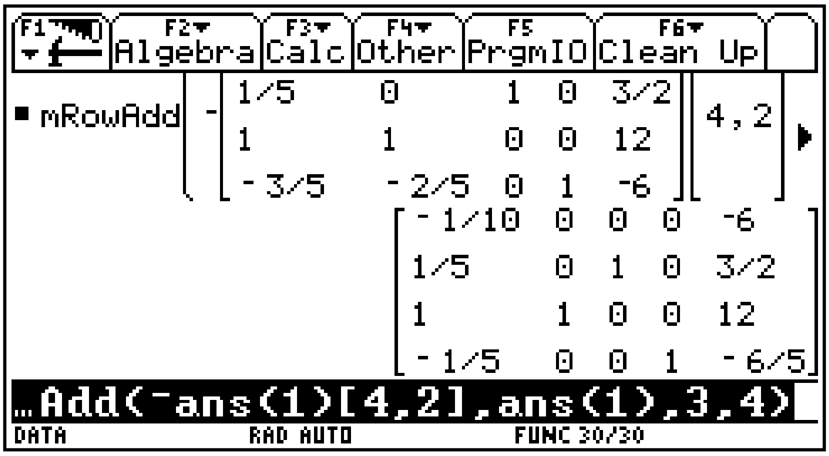
Dual simplex Method In the original set of constraints we add slack variable to the first, subtract surplus variable to the third and change the signs in the third constraint, this give us:

$$\begin{aligned} \text{Maximize} \quad & -Z = -\tfrac{2}{5}x_1 - \tfrac{1}{2}x_2 \\ \text{Subject to} \quad & \tfrac{3}{10}x_1 + \tfrac{1}{10}x_2 + x_3 \qquad = \tfrac{27}{10} \\ & \tfrac{1}{2}x_1 + \tfrac{1}{2}x_2 \qquad\qquad = 6 \\ & -\tfrac{3}{5}x_1 - \tfrac{2}{5}x_2 \qquad + x_4 = -6 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

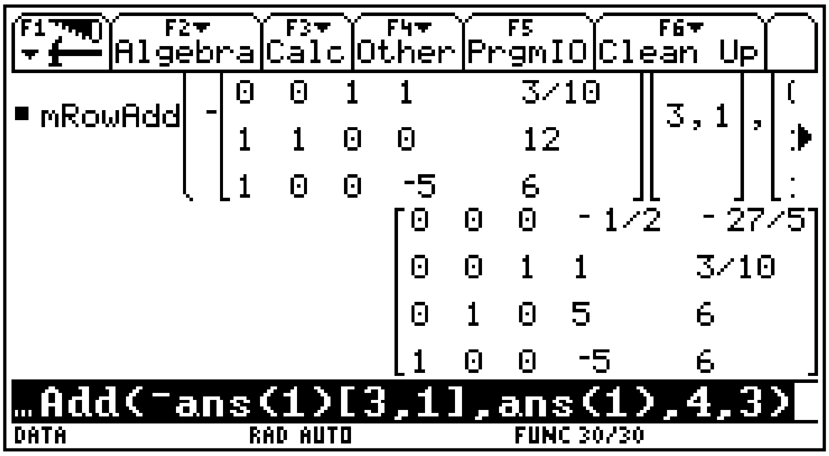
The corresponding matrix, in the calculator, looks like:



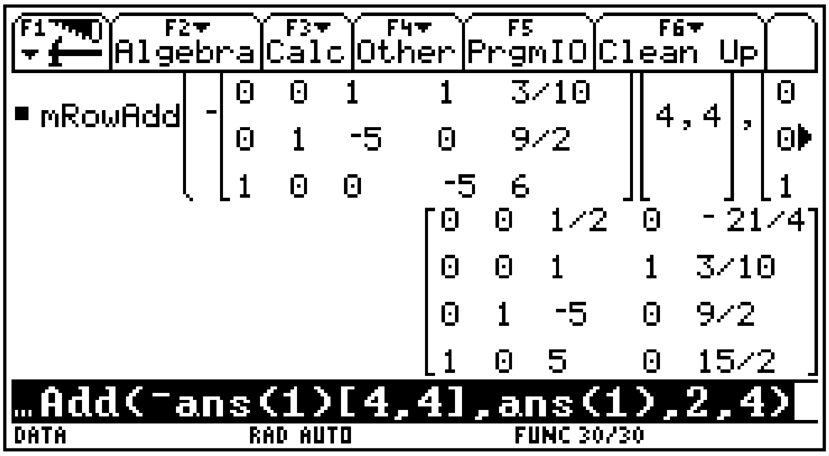
This, however is not a standard simplex tableau, we need a third basic variable. We choose x_2 in the second constraint, make its coefficient equal to one and zeros for the rest in its column.



The current solution is not feasible, we apply dual simplex to the last constraint to obtain



This is not optimal, it needs a regular simplex for the fourth column



This corresponds to an optimal tableau, and the solution is:

$$-Z = -\frac{21}{4},\ x_3 = \frac{3}{10},\ x_2 = \frac{9}{2},\ x_1 = \frac{15}{2}$$