STIMULATING SIMULATING WITH THE TI-83+: A LESSON FROM A MATHEMATICS COURSE FOR PRESERVICE TEACHERS

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Introduction: According to the National Council of Teachers of Mathematics (2000), technology is an essential tool in teaching and learning mathematics; not only does it influence the mathematics that is taught but it has the potential to enhance student learning (p. 24). In the ideal K-16 mathematics classroom, every student has access to technological tools which, along with a skillful teacher, help to facilitate his or her learning. In my role as a mathematics educator I strive to model appropriate uses of technology in the mathematics classes I teach for prospective teachers. In doing so, I hope to convey to them a sense of the versatility and power of technology along with guidance on how these tools may be used responsibly. In this paper I will provide an overview of how the TI-83+ graphing calculator can be used to investigate concepts in probability in a mathematics course designed for preservice mathematics teachers.

Context: Millersville University (MU) is a mid-size comprehensive public institution located in Lancaster County, Pennsylvania. Founded in 1855 as a normal school, teacher education remains a major focus of the curriculum. Presently, one-third of the approximately 7500 students enrolled at MU are in teacher education programs. All preservice elementary education and special education students at MU are required to take two three-credit mathematics courses specifically designed for this population. Presently, the required textbook for the two courses is Billstein, Libeskind and Lott's *A Problem Solving Approach to Mathematics for Elementary School Teachers* and the topics covered include units on number and operation, algebra, geometry, measurement, probability and data analysis.

Content—Probability and Data Analysis: One of five content strands in the NCTM's Professional Standards for School Mathematics (2000), probability and data analysis is assuming a more prominent place in the elementary and middle school mathematics curriculum. A quick perusal of three of Project 2061's (http://www.project2061.org/tools/textbook/matheval/default.htm) highest rated middle school (i.e., grades 5-8) mathematics texts, namely Connected Mathematics, Mathematics in Context, and Math Scape, revealed that probability concepts are featured significantly in each text. Further, many state-level mathematics tests (e.g., Pennsylvania System of School Assessment (PSSA)) now assess elementary and middle school students' understanding of probability concepts. Given the increased emphasis on this area of mathematics in the K-8 mathematics curriculum it is imperative that prospective elementary school teachers develop a profound understanding of the big ideas in probability. This task is especially difficult for several reasons. First, while most prospective teachers have a strong grasp on number and operations concepts they often lack experience with probability. Further, while computation of probabilities can appear to be simple work with fractions, there are often conceptual challenges involved in understanding probability. Finally, as Shaughnessy (1992) found many people hold misconceptions about basic probability concepts—misconceptions that are highly resistant to change. For example, individuals often believe that when tossing a fair coin, after a sequence of three heads the chance of getting a tail on the fourth toss is more likely than getting a head. As another example, students often believe that in a family of six children the birth order BGGBGB (where B represents boy and G represents girl) is more likely than BBBGGG since the first birth order appears to be "more random".

Using Monte Carlo Simulations to Teach Probability to Prospective Teachers: To help students uncover and resolve misconceptions in probability, NCTM (2000) recommends that students engage in solving real world problems that allow them to make predictions and to test their hypotheses by conducting experiments. Frequently individuals in business or industry want to know how a "system" will behave. If the system is fairly simple, it is often easy to use mathematics to determine how it will behave. However, as the system becomes more complicated, the equations may be too complex to solve or the mathematics may not have been invented yet. At this point it is often necessary to conduct a series of experiments to determine how the system would behave under different circumstances. Unfortunately, this is not always practical or possible as the experiments may be very expensive in time or money or may be too dangerous. In these cases, a series of *simulated* experiments can often be run—experiments which behave like the real system, but which do not have the disadvantages of the real experiments.

Using computer or graphing calculator simulations in the mathematics classroom affords students access to relatively large samples that can be generated quickly and modified easily. As noted by NCTM (2000) simulations with technology can facilitate students' learning of probability in two ways. First, with large samples the sample distribution is more likely to be "close" to the actual distribution, thus reducing likelihood of incorrect inferences based on empirical samples. Second, with easily generated samples, students can focus on the analysis of the data rather than be distracted by the demands of data collection. One well-known problem that I have used with preservice teachers to explore probability concepts via simulation is the Chinese Population Problem.

Chinese Population Problem:

> Instituted over two decades ago China's *one-child policy* is an extreme measure meant to control population growth. China's policy is usually misunderstood to mean all couples must have one child. However, the policy varies depending on where a person resides. In cities, the policy

restricts most couples to one child. Outside cities, a daughter exception is common — that is, a woman is allowed to have a second child if her first child is a girl. What impact has this policy had on the population? By the year 2000, the normal 107 boy to 100 girl birth ratio had shifted dramatically to about 117 boys to 100 girls. In rural regions, the difference reached levels as high as 130 to 100

Suppose you are the Health, Education and Welfare minister of China. The country is trying to decide whether or not to replace the policy of having only one child per family to allowing having only *one son per family*. This means that each family will be allowed to have children until the first son is born. However, they can have a maximum of five children. As HEW minister you must advise the government what the ramifications will be if the new policy is initiated.

Make sure your report addresses the following questions:

(1) PRIOR TO THE EXPERIMENT: Describe your predictions regarding how this "one-son" policy would affect the population of China including how the total number of males and females would be affected as well as the average family size.
(2) What is the probability of having families of size 1, 2, 3, 4, or 5 children?
(3) The government can't allow an average family size of more than two children per family. Will this new policy allow more than that number?
(4) Will there be a disproportionate number of boys or girls in the population?

Also be sure to include the following information in your report:

- A tree diagram of the theoretical model of the experiment that includes the theoretical probability of each possible outcome of the experiment.
- A description of how you decided to model the experiment based on the theoretical probabilities.
- A report of the results of your simulation using the model you decided upon.
- A comparison between what you would have predicted would happen based on the theoretical probabilities and what actually happened as a result of the simulation.

Helpful Hints for Using the TI-83+ to Solve the Chinese Population Problem: Using the random number generator on the TI-83+ it is easy for students to simulate and solve Chinese Population Problem. Assuming that having a boy or a girl are equally likely events students can assign the random number 1 to "girl" and 2 to "boy" in the following way.

To assign random numbers to the events of having a girl or boy:

- Turn calculator on
- Press MATH Key (third key, left column)
- Slide to PRB using blue cursor key
- Press 5
- Type in 1, 2, 1 as follows: randInt(1,2,1)
- (comma is in column 2, parentheses next to it)
- Press ENTER
- Again
- Again

To store these data and allow for further manipulation of the data:

- Turn calculator on
- Press MATH Key (third key, left column)
- Slide to PRB using blue cursor key
- Press 5
- Type in 1, 2, 1 as follows: randInt(1,2,1)
- Press STO ➡
- Press 2nd, 1 (stores data in L1)
- Press ENTER

To perform simple statistical analysis of the data:

- Press STAT
- Slide to CALC
- Press 1
- Press 2nd, L1
- PressENTER

Summary: Using the TI-83+ to simulate the Chinese Birthday Problem has been effective in helping my students make sense of several concepts in probability (e.g., the difference between theoretical and experimental probabilities). My students find the Chinese Birthday Problem extremely engaging and quickly learn about the benefits of using the TI-83+ to simulate and solve problems requiring the collection and analysis of large samples of data. The students' initial hypotheses about the problem are revealing and they are often frequently surprised by the results they get via the simulation. In closing, the Chinese Birthday Problem, and several others like it, allow me to achieve the goal of helping my students—teachers in the making—come to a deeper understanding of mathematics while learning how technology can be effectively integrated in the mathematics classroom.

References:

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