

WORKBOOK EMULATION: THE SCANDALOUS STATE OF MATH SOFTWARE

John C. Miller

Department of Mathematics, The City College of CUNY

110 Riverside Dr. #14C; New York, NY, 10024

xyalgebra@mindspring.com

A. Definitions and Axioms

A math problem is **non-trivial** if it admits at least two significantly different methods of solution. Although "significantly different" is subjective, the intent should be clear. Alternative methods of solution of non-trivial math problems should be encouraged and supported.

Some students attend classrooms with instructors, some work from a textbook or workbook, and some use tutorial software. Regardless of the environment, at any given time during the learning process a student is in one or the other of the following learning modes:

1. In **presentation mode** the student's instructor, textbook author or software presents new material. Presentation mode includes showing solutions to sample problems. In presentation mode the student is usually comparatively passive.
2. In **problem solving mode** (the name is self-explanatory) the student is highly active.

The following are axiomatic:

1. For most students, most of the learning occurs in problem solving mode, rather than in presentation mode.
2. A student who, in problem solving mode, attempts to solve a non-trivial problem in a non-standard way should always be encouraged and supported.

B. The Scandal: Settling for Workbook Emulation

A textbook or workbook author cannot realistically support a student who attempts to solve a problem using a non-standard method. A classroom instructor can **sometimes** find ways to provide such support. Instructional software can **always** be designed to provide intelligent, step-by-step support to all students in problem solving mode.

Unfortunately, most current commercial math software relies on multiple-choice, matching or short answer formats. A "canned" solution is sometimes displayed later. This problem-solving support model actively discourages students who use even slightly non-standard methods to solve non-trivial problems, especially when they don't obtain

exactly the correct final answer. It closely emulates a workbook, and provides no more help than a workbook.

These answer formats convey insufficient information to the computer to permit it to offer the student intelligent help. In fact, intelligent help should be available not only after the student has entered a final answer but also earlier, as the student works toward a solution. The student should be able to enter intermediate steps and the computer should respond intelligently to each step by accepting it as correct or rejecting it, suggesting why it was rejected and then, if asked, suggesting a reasonable next step. Of course, this requires that the computer be programmed to implement intelligent mid-problem changes in its own solution strategy.

The real scandal is that students who are clever, courageous or ambitious enough to attempt to solve problems using non-standard methods are most often those who will make the best use of their mathematical training, either in more advanced mathematical work or in another area where mathematics is applicable. These are the students on whom the future depends. It is scandalous not to support them intelligently.

C. Intelligent Step-by-Step Help Is Not That Hard

Artificial Intelligence techniques have been under development for at least 40 years. In fact, only elementary techniques from computer science are required in basic mathematics.

Example 1. Is the Student's Step Correct? With probability 1, two rational expressions in n variables are equivalent (and so replacing one by the other is acceptable when simplifying) if and only if they have the same numerical value at a single randomly chosen point in R^n . The proof uses the fact that the probability that a randomly chosen point in R^n lies on a given hypersurface in R^n is 0. Numerical evaluation of expressions is trivial by using their expression trees. Any competent computer science major could write the needed routines.

Example 2. Suggesting the Next Step. A program can intelligently suggest the next step in an expression manipulation problem by analyzing the expression tree of the current step. It should seek places to apply the basic simplification rules of algebra. These include the field axioms and, depending on the level of the student, other basic rules such as "invert and multiply" and $(a^b)^c = a^{bc}$. The program should then use the context and the student's current level of sophistication to choose which of several applicable rules is the most reasonable one to apply. After applying this rule to the current expression tree, the program can suggest the expression corresponding to the revised tree as the next step, and describe it in terms of the selected rule.

For more details and additional examples, see "Intelligent Help at Every Step: Critically Important, Surprisingly Simple Yet Distressingly Rare" by John C. Miller in *The Proceedings of the Fifteenth Annual International Conference on Technology in Collegiate Mathematics*.

D. What Can Be Done?

The blame that so little problem solving support is available in current computer-based instructional materials lies with the thousands of faculty members who participate in the process of evaluating and adopting materials for their institutions' students. The requisite software is undeniably more difficult and expensive to produce than the current "short answer and canned solution" packages. Publishers will produce it only when significant numbers of faculty educate themselves as to what is needed and what is feasible, and then settle for no less for their students.

For an example, go to www.xyalgebra.org. Click on "Intelligent Help" for a simulation involving solving an equation. Download and run the "6 Minute Demo" to witness intelligent step-by-step help in the context of a verbal problem. The xyAlgebra package is an academic and completely non-commercial project, so both the tutorial course materials and the instructor's database manager can be downloaded and used by students and instructors at no charge.

Two commercial sites offering materials with intelligent help are: www.mathkal.co.il and www.mathxpert.com.