CAN TECHNOLOGY HELP PREDICT THE DYNAMICS OF STUDENT ATTITUDES TOWARD MATHEMATICS?

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This paper addresses research on the dynamics and development of college student attitudes toward mathematics (SATM). The theoretical framework of the study is based on the mathematical ideas and techniques of strange attractors and chaos and on numerous examples of their application to the hard and social sciences. The empirical component of the study is based on the dynamics of SATM measured by a Likert type attitude scale in one of the Indiana University's mathematics classes for pre-service teachers each day of an entire semester. The purpose of the empirical study is to identify and analyze specific mathematical patterns of strange attractors and chaos (i.e., irregular, chaotic, complex pattern) in the dynamics of SATM and interpret them in terms of such educational parameters as SAT/ACT scores using the SPSS graphs and outputs.

The last two decades have been characterized by the increasing popularity of strange attractors and chaos in the hard sciences, biology and medicine, which inspired social and political scientists' growing interest in this new paradigm (Grebogi & Yorke, 1997). Due to its highly interdisciplinary character (Feigenbaum, 1997), the concept of strange attractors provided a framework of ideas to researchers in different natural sciences and sociology for organizing their thoughts and observations. This framework allowed them to understand and explain complex nonlinear phenomena in their respective fields of investigation (Ding, Grebogi, & Yorke, 1997; Houghton, 1989). As a result, the strange attractors and chaos epistemology became an umbrella for new scientific theory in a very diverse family of disciplines from biochemistry and anthropology to political science and family therapy (Butz, Chamberlain, & McCown, 1997; Campbell & Mayer-Kress, 1987; Mayntz, 1997). The interdisciplinary character of the strange attractors and chaos paradigm, as well as its features of ubiquity and universality discovered in different fields of knowledge, lead to the possibility of using strange attractors for understanding the dynamics of SATM.

The theoretical component of this study is designed as a re-specification and conceptual interpretation of the notions, methods and ideas related to the strange attractor paradigm in terms of SATM. This design also includes a replication and re-specification of specific mathematical models and differential equations representing strange attractors and chaos in political science, biology and physics.

In particular, one of the examples of turbulence in the idealized dynamics of SATM might be modeled through re-specification of a Navier-Stokes type system of nonlinear differential equations

$$x' = ax + dxz$$
$$y' = by + eyz$$
$$z' = cz - fx^2 - gy^2$$

where the variables $x(t)$, $y(t)$ and $z(t)$ represent three different dimensions of SATM such as enjoyment, frustration, and confidence. This system provides complex non-trivial patterns of behavior that can be represented theoretically by strange attractors with non-integer Hausdorff dimensions (Constantine & Foias, 1985). In fact, this system already suggests that such a relatively simple (in a psychological and educational sense) idealized model of dynamics of student attitudes toward mathematics provides real, "scientific," chaos that is extremely difficult for mathematical analysis. Given that this model is psychologically and educationally idealized and oversimplified and represents only three factors of student attitudes toward mathematics with only seven correlating parameters, one can imagine that the real picture with *hundreds* of interacting psychological and social factors and parameters would be enormously complex. Besides that, this example might provide a rationale for the hypothesis that chaos and strange attractors are not just an abstract mathematical creation, but can somehow represent real patterns of student attitudes toward mathematics. This rationale might be expressed, in particular, by the following hypothetical question: if strange attractors and chaos are found already in oversimplified idealized models of dynamics of student attitudes toward mathematics, then why can't strange attractors and chaos exist in real models of such dynamics?

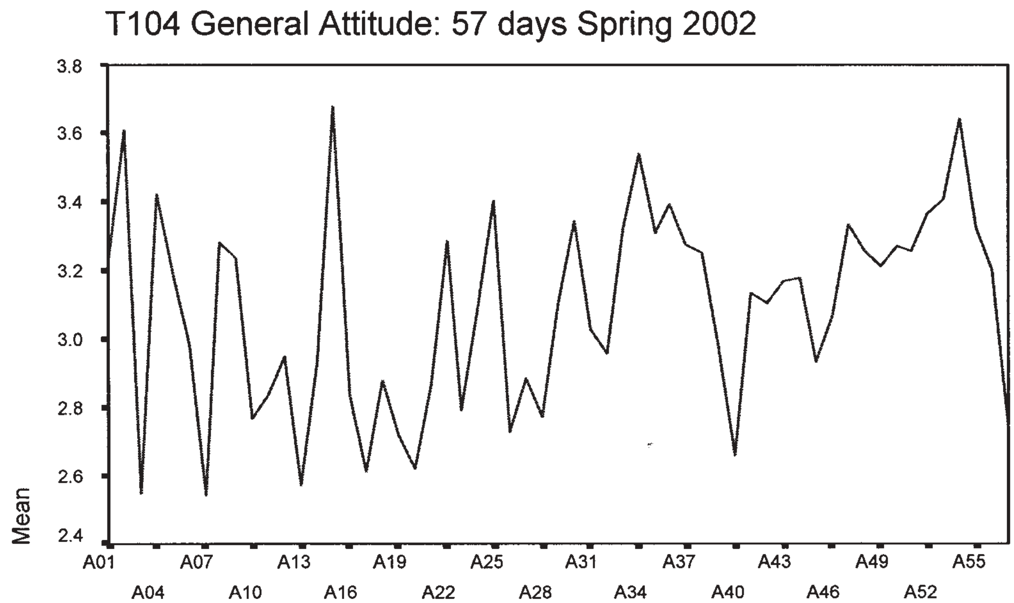
The empirical component of the study is focused on the dynamics of SATM in one of the T104 problem-solving mathematics classes for preservice elementary teachers at Indiana University. The same Likert-type scale of attitudes toward mathematics consisting of six items was administered to all 29 students enrolled in the class four days a week during the entire spring semester of 2002 (57 days total). Each item represents one of the following dimensions of SATM: enjoyment, confidence, boringness, importance for life, frustration, and importance for teaching (see Table 1). This scale has a reliability alpha of .84 with item-total correlation coefficients between .54 and .73, and is based on a larger scale consisting of 36 items (alpha .92) administered in three different sections of T104 during a pilot study in fall 2001.

Table 1
Mathematical attitudes towards T104

1. I didn't enjoy today's T104 session.
2. I know I did well in T104 today.
3. Math problems solved in T104 today were boring for me.
4. The math topic we covered in T104 today was very important for the everyday life.
5. I was confused in T104 today.
6. It was easy to see how today's T104 lesson will make me a better teacher.

At the end of the semester, based on the SPSS outputs, all the aggregate levels of mathematical attitudes for the whole scale and the six sub-scales were graphed in order to demonstrate any irregular or chaotic patterns visually. All of these SPSS graphs represent complex chaotic and irregular dynamics of the T104 students' attitudes toward mathematics (one of the graphs is shown on Figure 1).

Figure 1

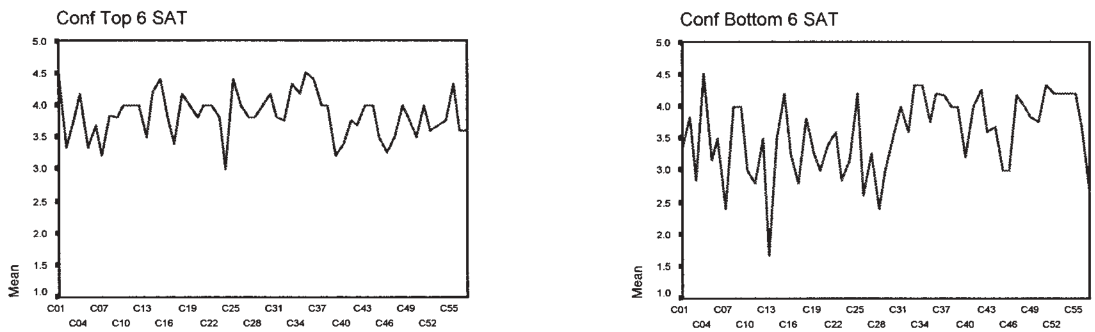


This dynamic pattern presented in Figure 1 somewhat resembles (in terms of chaos and irregularity) those patterns that were hypothetically described by strange attractors in some other interdisciplinary studies mentioned at the beginning of this paper. Among such dynamical patterns are the 7-week psychological dynamics of "Symptom Change in the Family through Time" (Butz et al., 1997, Figure 10.2, p. 209), as well as the 31-week dynamics of "Total Symptom Scores of Family in Treatment" (Butz et al., 1997, Figure 11.2, p. 231) in family therapy. Besides that, visually similar complex and irregular patterns can be found in the dynamics of brain electroencephalogram rhythms (Mandell & Selz, 1997, Figure 13, p.91) and the output patterns of the chaotic neural network (Aihara, 1997, Fig. 10, p. 123). Kendall et al. (1997) report similar patterns in the dynamics of monthly reported cases of measles in three North American cities (Figure 4, p. 201) and in the electroencephalogram trace of an epileptic seizure (Figure 10, p. 207).

Furthermore, Figure 2 demonstrates that the dynamics of confidence for the top SAT group of students fluctuates less than the confidence for the bottom SAT group. ("Less fluctuation" here means that the range and frequency of the fluctuations/oscillations are smaller, which makes the whole graph and dynamics relatively more "smooth" and stable, and "less" chaotic and irregular.) This observation suggests that SAT/ACT math score might be the parameter that explains bifurcations in the dynamical system of

SATM. This also might mean that the observed differences among the identified categories of students could be used for diagnosis of patterns of anxiety and other emotions about mathematics with respect to such parameters as SAT/ACT math score, GPA, or course grade.

Figure 2
Confidence for Top and Bottom SAT Groups



One of the most general theoretical findings of the study presented in this paper is the philosophical understanding that non-linearity and irregularity might be not a result of measurement errors or random noise, but the very nature of evolution of SATM. This is especially important for the methodological understanding that the traditional statistical methods and epistemology might be irrelevant to modeling and analyzing dynamic processes of changing student attitudes. Student attitudes are a specific case of more general social and human phenomena, which represent, according to Butz et al. (1997), Houghton (1989), and other authors, complex nonlinear and irregular processes. Since in the past these nonlinear processes were often inappropriately modeled by traditional statistics based on the linear paradigm of cause and effect, the authors called for a large-scale methodological shift to the strange attractor and chaos paradigm as a more appropriate framework for understanding complex social and human phenomena. The introduction of this paradigm is the main methodological aspect of the study presented in this paper that helps to overcome the problem of the likely nonlinear nature of the SATM and the linear character of traditional statistical methods of analysis.

The introduction of the strange attractor and chaos paradigm for student attitudes also provides a number of more specific theoretical findings that can be expressed in terms of such theoretical constructs as irreversibility, complexity, non-linearity, instability, unpredictability, self-organization, and irregularity. In particular, according to this paradigm, SATM are highly sensitive to initial conditions ("the butterfly effect"): a tiny change in attitudes today can result in global structural changes in the system of attitudes in the future. SATM may evolve and qualitatively change their behavior through sequences of bifurcations associated with variation in parameters of the system (e.g., SAT/ACT scores, GPA, course grades). Individual trajectories of SATM are generally unpredictable for the long-term future due to their high instability (sensitivity to initial conditions), complexity and strong non-linearity. At the same time, due to the structural stability of attractors, slight measurement errors are not crucial or significant with respect to far future dynamics of attitudes for whole student populations in general.

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