CIRCUITS, TANKS, AND MATHEMATICAL MODELS

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Introduction

Many of the engineering and science students who pass through standard Ordinary Differential Equations (ODE) course remain unconvinced of the utility and accuracy of the simple models presented in their ODE texts. Straightforward experiments demonstrating the quantitative accuracy of the models have the potential to improve such technical students' view of the utility of mathematics. This paper demonstrates high quality computer data collection for standard electric circuit and tank mixing problems modeled in ODE courses. The experiments and analysis use inexpensive equipment and standard software. In addition, the experiments are designed so that they can be performed in front of a large class without consuming excessive lecture time in what is commonly a course with a tightly packed syllabus. The collected data validates standard ODE model predictions with surprising accuracy for most students and mathematics faculty members. The hope is that improving science and engineering student opinion of the utility of mathematics in what is for many their capstone mathematics course will motivate further study of mathematics.

There is a huge difference in a student's mind between qualitative agreement with reality

"This looks as though it agrees pretty well!"

and quantitative agreement with reality

"This agrees with an error of less than 1%!"

We can and should teach undergraduates to think critically about the connection between the mathematical models in calculus and ODE and physical reality. We can do our science and engineering colleagues a favor by convincing students that simple standard models in math classes are standard because of striking quantitative agreement with physical reality.

Student Attitudes To ODE Models

Surveys we have conducted in ODE classes at MTU indicate that many science and engineering students in sophomore/junior ODE classes do not believe that the models they study in math classes provide more than qualitative agreement. This is not surprising when you realize that undergraduates really only see qualitative comparisons between models and real world data!

The MTU surveys suggest that the treatment of modeling in current courses and texts is inadequate. Specifically

- Current courses do not convince undergraduates of the utility and ubiquity of the ODE models in science and engineering.
- Current courses do not demonstrate the quantitative accuracy of ODE models in science and engineering.

These surveys prompted a continuing project to develop inexpensive, convincing, scientifically sound experiments to convince students of the utility and accuracy of ODE models. The questionnaires (distributed to several hundred students in several sections of our engineering ODE course) indicated that roughly:

- One third of the students believe the models they were being taught were grossly inaccurate.
- Another third of the students do not understand what accuracy means.
- The remaining third do not realize how accurate standard ODE models are.

A careful survey of the effects of experimental data collection base course materials on our student's beliefs is planned.

To be widely used such experiments must be:

- Easy to set up and fast to do.
- Hard to mess up and convincing.
- Inexpensive.

The experiments also need a planned set of activities and student worksheets, ranging from simply doing some minimal data processing to conducting and analyzing modified experiments.

Common Mathematical Models in ODE

The two common engineering models in ODE texts that provide the simplest and best comparison with experiments are tank models and circuit models. We focus on circuit models and demonstrate some undergraduate-developed projects to use the audio-in and audio-out capabilities of modern computing equipment to conduct simple, inexpensive, and convincing circuit experiments.

There are three basic linear circuit elements Resistors, Capacitors, and Inductors. The underlying equation for the simplest circuit containing all these elements along with a voltage source is an LRC circuit presented and discussed in over 90% of introductory ODE texts. The governing ODE is

$$L\, q''[t] + R\, q'[t] + q[t]/C = E(t).$$

L, R, and C are constants while E(t) is a specified voltage source.

The above, along with a circuit diagram, and a very brief discussion of Kirchoff's Laws is pretty much what gets presented in most ODE text.

There is no mention that:

- The three linear circuit elements (Resistors, Capacitors, and Inductors) are all available incredibly inexpensively in a range of values (Ohms, Farads, and Henries) at your local Radio Shack.
- The values L, R, and C are simply read (with tolerances) from the side of the package.
- Almost all circuits contain only these linear elements and two nonlinear elements (diodes and transistors).
- Commercial circuit design software simply numerically simulates a large system of ODEs composed of these five elements.

Traditionally to conduct a circuit experiment in a Math department you would need to purchase:

• A signal generator	~$9,000	
• An oscilloscope	~$2,000	(cheap no data storage)
	~$9,000	(digital data storage)
• A breadboard	~$20	(fancy with plugs)
	~$5	(non-fancy)
• Circuit components	~$10	(total)
• Computer	already available	
• Data Transfer	already available	
• Analysis Software	already available	

Using the audio-out as a signal generator and the stereo audio-in as a data collection device to perform the same experiments on a PC with a Computer Algebra System (CAS) you need to purchase:

• A breadboard	~$20	(fancy with plugs)
	~$5	(non-fancy)
• Circuit components	~$10	(total)
• Computer	(already available)	
• CAS	(already available)	
• External USB soundcard	~$50	(optional)

So the PC based version of the experiments can be performed for an additional cost of less than $50.

Additional benefits of the PC version of the experiment is that it is simpler:

- The only specialized equipment is the circuit board and components.
- The signal generation is straightforward and explicit.
- The data collection is straightforward and direct.

- The analysis is automatically quantitative since the data is imported into a CAS.
- The presentation of the experiments is integrated into the analysis since they are naturally conducted in the same CAS document.
- The experiments are fairly easy and rapid to conduct in a large lecture section.
- Students can conduct their own experiments since the additional equipment required is inexpensive and difficult to damage.

There are some inexpensive handheld data loggers available which can collect data from an electronic circuit. The most common is the Calculator Based Laboratory (CBL) that provides an interface between a Texas Instruments (TI) calculator and a variety of measurement probes. Although the CBL is widely used we claim the PC and CAS have numerous advantages:

- The CBL only collects the data. Data analysis still needs to be performed separately.
- The CBL needs to be programmed.
- The maximum sample rate of the CBL is significantly lower than the 164000 samples per second available on most PC.
- The constrained memory of the CBL restricts the amount of data that can be collected.
- With the PC the analysis and signal generation tools are integrated with the presentation.

Experiments

The ease with which the experiments can be conducted is easily demonstrated. An RC (Resistor and Capacitor) circuit was set up during the presentation and the amplitude ratio and phase shift measured and compared to the theoretical predictions from the ODE at a number of frequencies. The measured and predicted values of the amplitude and phase shift agreed to within 1%. The experiment was repeated with a LR (Inductor and Resistor) circuit with similar results.

Completing the Project

The complete project consists of:

- A standard RC and LR circuit steady state analysis for a fixed frequency.
- A standard RC and LR circuit steady state analysis with a frequency sweep.
- A resonance demonstration in a LRC circuit with a frequency sweep.
- A complete collection of DC experiments.

Numerous supplemental projects are planned:

- A suite of experiments to measure the resistance, capacitance, and inductance of linear circuit elements.
- A suite of experiments to measure non-linear circuit elements such as diodes and transistors.
- A suite of experiments with "interesting" nonlinear circuits.
- A brief tutorial describing the uses of the Fast Fourier Transform to analyze periodic signals.

Conclusions

The audio input and output of modern computing equipment enables accurate experiments on simple electric circuits that in the past would have required expensive lab equipment. We believe:

- Experiments of this nature can enhance student understanding and retention of the simple circuit models in the ODE course.
- Generate data to help convince students of the accuracy of ODE models.
- Help link required math courses with material from other disciplines.