# A WEB COURSE FOR IN-SERVICE HIGH SCHOOL MATHEMATICS TEACHERS

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In the spring of 2002 this author developed and implemented a web-based, graduate level mathematics course for in-service high school mathematics teachers pursuing the degree Master of Arts in Mathematics from the Department of Mathematics and Statistics at Texas Tech University. A discussion of the development, implementation, and assessment of this course was presented by this author at the 2002 Vienna International Symposium on Integrating Technology into Mathematics Education and published in the symposium proceedings.[1] The course is being offered for the second time during the fall of 2003. Herein we briefly discuss the rationale for the development of the course, the goals of the course, the course content, and the logistics involved with the distance delivery of the course. Also we provide examples of the course materials.

**Rationale.** We believe it is important for in-service teachers at all levels, in all subjects, to be willing and able to continue learning. In the age of super graphing calculators and computer algebra systems this seems especially true for teachers of mathematics. However the professional demands of the job make it difficult for working teachers to take graduate courses while their own schools are in session, and practically impossible if taking such a course would require a lengthy commute to a university campus. Thus it is imperative that universities located, as is Texas Tech, in the center of large and relatively rural geographic areas, explore alternatives to the traditional classroom delivery of graduate level mathematics courses. One such alternative is offering courses via the internet, without the students having to physically appear on campus. Such is the course we describe below.

**Goals.** We have two primary goals. One is to enhance the teachers' understanding of the wide range of mathematical concepts encountered in their own classrooms. The other goal is to produce competent and confident users of mathematics-specific technologies such as computer algebra systems and super graphing calculators. Additionally we wish to produce teachers who are willing and able to adjust to the ever-evolving technology and make appropriate decisions about its use in their classrooms. Finally, we want to achieve these goals without the students having to commute to our campus.

**Content.** We focus on mathematical content appropriate for the high school level. The course consists of three chapters required of all students. Chapter I is called Arithmetic with MAPLE and consists of three sections. Section I.1 covers basic calculations and stresses exact versus floating point approximate arithmetic. Section I.2, Elementary

Number Theory, deals a lot with prime numbers and consequences of the Fundamental Theorem of Arithmetic. Section I.3 covers the Binomial Theorem and Pascal's Triangle.

Chapter II consists of two sections and is titled Algebra with MAPLE. Section II.1 deals with the Fundamental Theorem of Algebra and issues involving rational, real, and complex roots of polynomials with real coefficients. Chapter II concludes with a section on graphing, with a lot of time spent on graphs of rational functions, trigonometric functions, conic sections, and 3-d graphs.

Calculus with MAPLE is the topic of Chapter III. Limits, Derivatives, Integrals as Area, Integrals as Anti-derivatives, and Sequences and Series are covered in five sections. There is heavy emphasis on the use of the graphing capabilities of MAPLE for the purpose of demonstrating these concepts.

For the remainder of the course the students choose one of two possible options. They can elect to complete either a chapter on programming with MSW LOGO or a chapter on Linear Algebra. The linear algebra chapter covers matrices, linear systems, vector spaces, eigenvalues, and eigenvectors.

**Delivery.** Our course is delivered over the internet using the commercial web tool WebCT. In addition to access to the internet, the students are required to have access to MAPLE, currently version 9. (The student version of which is available from our campus computer store for $120.) Those who opt to complete the LOGO chapter must also obtain MSW LOGO, which can be downloaded free from the internet (http://www.softronix.com/logo.html).

Students download the material for Chapters I, II, III, and V from the course WebCT site in the form of MAPLE worksheets. The LOGO material in chapter IV consists of WORD files. They complete the exercises on the worksheets and submit them to the instructor as e-mail attachments using the WebCT mail utility. The instructor opens the worksheets, evaluates them, adds comments and suggestions as needed, then returns the worksheets to the students via the same mail utility. To pass the course students must complete all worksheets to the instructor' satisfaction. Final letter grades are assigned to each student based on his/her performance on practice quizzes (5 and a project for students choosing the LOGO option, 7 for students choosing the linear algebra option), two midterm exams, and a final exam. The quizzes and exams are in the form of MAPLE worksheets and their submission and evaluation is handled the same way as all other work.

Deadlines for completion of student work are posted on the WebCT course calendar with quizzes and exams being released to the students at appropriate times prior to the deadlines for their completion. Keys to all quizzes and exams are released when all students have completed them.

In addition to the continual communication between students and instructor afforded by the course mail utility, the WebCT chat room utility is used to conduct a regularly scheduled virtual class. The students and the instructor are all expected to logon to the course chat room for a specified 3-hour time period once per week. (In the current session this is Monday from 4:00 pm until 7:00 pm CST.) Most of this time is spent with the instructor responding (in real time) to student questions about exercises on the worksheets. Some small amount of time is occasionally spent with the instructor lecturing over something that appears to be unclear, or confusing in a worksheet. The ability to copy and paste from and into an execution line of a MAPLE worksheet via the chat room is very helpful in these discussions. Of course it helps to be able to type.

**Materials.** The following sample exercises are from Chapter I. From I.1 comes

> Use both MAPLE and your calculator to compute $1/50!$ and $1/50!+1$. Compare the results and make appropriate comments.
>
> Using both MAPLE and your calculator try to convert $\sqrt{2}$ to an equivalent fraction form. What is your conclusion? Explain why the following MAPLE command sequence produces a result that might be misleading: convert(evalf(sqrt(2),fraction) .

Section I.2 contains a discussion of the Fundamental Theorem of Arithmetic. After looking at a lot of examples of factorizations of pairs of integers along with factorizations of their gcd and lcm, the students are give the following exercise:

> Using the FUNDAMENTAL THEOREM OF ARITHMETIC, provide a written argument to verify the formula gcd(m,n)lcm(m,n) = mn for any two positive integers m and n.

Chapter I concludes with a discussion of the Binomial Theorem and Pascal's Triangle. The following is a sequence of exercises from section I.3:

> State the BINOMIAL THEOREM and use MAPLE to demonstrate it.
>
> Compute $\sum_{k=0}^{n} binomial(n,k)$ for n=0 to 10.
>
> What general formula does the above calculation suggest?
>
> Use MAPLE to verify the formula for all n.
>
> Use the BINOMIAL THEOREM to "prove" the general formula.

Chapter II covers properties of polynomial and rational functions and graphs of them. A lot of emphasis is placed on the FUNDAMENTAL THEOREM OF ALGEBRA. The

MAPLE commands most used are solve( ), fsolve( ), and factor( ). Use of the factor command quickly leads to a discussion of factorization over the rational, real, or complex number field. At one point in section II.1 the students are given the following instructions:

> For the polynomials $p_5$ through $p_{18}$, solve for all exact roots where ever possible, find all approximate roots to 10 decimal places and compare with the results you get from the poly solver (or solve) package on your calculator. Finally factor each into linear factors. (Hint: The solve and factor commands default to looking for roots over the rational number field. To find roots and factors over the real or complex fields we sometimes need to add the optional command real or complex respectively to input in the solve, fsovle, or factor commands. You can check Help for these commands to see just how to do this.)

One such poly is the following: $2x^4 + x^3 + x^2 + 1$. The students are asked to plot the graphs of the polynomials and asked to explain if the graphs they are observing are consistent with the calculations of the roots. After these examples they are given the following exercise:

> Explain why every odd degree polynomial must have at least one real root, while an even degree polynomial can have no real roots at all.

Students are provided the function $f = \dfrac{2x^5 + 5x^4 + 6x^3 + 6x^2 + 4x + 1}{4x^5 + 18x^4 + 24x^3 + 24x^2 20x + 6}$ and asked the following series of questions:

> What are the x-intercepts of f?
> What is the y-intercept of f?
> What are the vertical asymptotes of f? (Try plotting the graph of f to see this.)
> Why does f have only one vertical asymptote when its denominator has 3 real roots?
> Is the vertical line really part of the graph of f? Explain.
> Provide a graph(s) of f that clearly exhibit the relevant behavior of f.

Section II.2 explores the graphing capabilities of MAPLE with the following exercises being typical:

> Provide a graph that clearly demonstrates the effect of changing the parameter k in the following relation: $x^2 + ky^2 = 1$.

> Provide a graph of the surface given parametrically by $x = s + t$, $y = s^2$, $z = t^2 + s$. Do you think we should call this a "smooth surface?" Explain.

Chapter III covers the standard topics from the Calculus. The following is a question on limits from III.1:

Let f be the function $\sin(1/x)$ if $x \neq 0$ and 0 otherwise. Plot the graph of f then use MAPLE to compute its limit as $x \to 0$. Explain the MAPLE output.

An exercise in Chapter III.2 on derivatives concludes by asking the students to plot the graph of the relation $x^3y + y^2 - x^2 + xy = 3$ and both the tangent lines corresponding to x==0. The section on integration via area exploits the MAPLE student package using commands like middlebox, rightbox, leftbox, and the corresponding sums to reinforce the concept of area as a limit of rectangular objects. Near the end students are asked to

Find the area of the finite region determined by the curves $x^3 - y^2 = 0$ and $x + y^4 = 2$.

In section III.3 the students are asked to perform a series of routine exercises intended to demonstrate the FUNDAMENTAL THEOREM OF CALCULUS. Chapter III concludes with an exercise using lefbox and rightbox to demonstrate that the series $\sum_{i=1}^{\infty} \frac{1}{i^p}$ converges if and only if $p > 1$.

The LOGO chapter covers basic programming through recursion and end with a project in which the students must write a program for constructing a picture of a school house on a starry night complete with a crescent moon and appropriate landscaping. Students choosing to complete the linear algebra material learn how to use MAPLE to do basic matrix operations, find eigenvalues and eigen vectors, and use matrices to solve linear systems. They also see a general demonstration of the Caley-Hamilton Theorem.

**Conclusion.** Our experience suggests that these materials are effective in achieving some of the goals indicated above. However, using them is quite labor intensive, especially in the distance delivery format. They require a great deal of perseverance and patience on the part of both the students and the instructor. We should mention that similar exercises along with examples of student responses can be found in a previous paper written by this author. [2]

## References


1. G. Harris, *Distance Delivery of a Graduate Level Mathematics Course for High School Teachers,* Proceedings of the Vienna International Symposium on Integrating Technology into Mathematics Education, Vienna Austria, 2002.
2. G. Harris, *The Use of Computer Algebra System in Capstone Mathematics Courses for Undergraduate Mathematics Majors,* The International Journal of Computer Algebra in Mathematics Education, 7 no 1(2000), 33-62.