

TANK TIME: STRUCTURED MAPLE PROJECTS FOR DIFFERENTIAL EQUATIONS

Joseph Fadyn
Southern Polytechnic State University
1100 South Marietta Parkway
Marietta, Georgia 30062
jfadyn@spsu.edu

BACKGROUND – THE GRANT

In 2000 the author and a colleague (M. Dillon) received a grant from the University System of Georgia to develop and implement a course in differential equations which would use technology—including, but not limited to, the TI-89 calculator and other computer algebra systems—in an essential way. As outlined in the grant proposal, the authors team-taught a technology intensive section of differential equations during Spring Semester, 2000. The grant called for the authors to make full use of multimedia presentations, the Web, the TI-89 and other computer algebra systems in classroom presentations.

LESSONS LEARNED

The team-teaching aspect of the course went quite well. The author's skills sets complemented each other nicely with Professor Fadyn taking the lead with the TI-89 and Maple and Dr. Dillon taking the lead with Power Point presentations and web-related issues. It was clear that the TI-89, although a valuable tool, was somewhat underpowered for this course and required considerable supplementing via user-defined functions and programs. It was also clear that a good source of projects which did not require students to have a substantial knowledge of Maple was needed. In addition, these projects needed to be original, since many projects out there are quite "well-worn" and indeed in a number of cases, solutions can be found published to the web and accessible to students.

FACULTY DEVELOPMENT AT GEORGIA INSTITUTE OF TECHNOLOGY

The author was accepted into the Faculty Development Program at Georgia Tech for the Fall Semester 2000 and the Spring Semester 2001. My proposal was to write a number of structured Maple projects for differential equations. These projects would be "structured" in the sense that they would be written in the form of a Maple worksheet which would guide the student through completing the project. This guidance would allow for a minimal knowledge of Maple by the student since in many (although not all) cases, examples of the Maple syntax would be blended into the

presentation and would precede the questions asked in the project. Other questions asked in these projects might, for example, produce a plot or an animation and require detailed explanations of these by the student. The idea was that students should learn some Maple, but not be overburdened by the Maple syntax since this would certainly take away from the main idea of using the technology to aid in understanding the mathematics.

BEYOND GEORGIA TECH

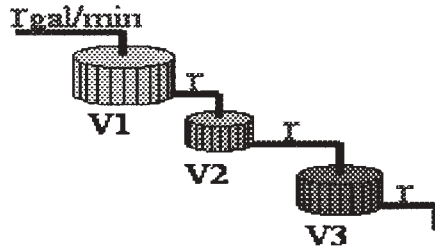
Using my experience at Georgia Tech as a springboard, I have written a number of additional Maple projects for differential equations. The projects were written in Maple V release 5 mainly because this is the version of Maple which is accessible to students at my home institution. However, for purposes of this talk, I have updated these projects to Maple 8. I have also tested them in Maple 9 (the “classic worksheet” version and they ran just fine). The disk which I will be distributing (described below) was written in Maple 8.

THE DISK

During my presentation at ICTCMXV, I will be distributing a disk which contains a set of three structured Maple projects for differential equations, as well as complete solutions to each project. Although I retain the copyright to this material, I hope that those who receive the material will be able to use some of it in their differential equations classes. I would ask, however, that you do not provide students copies of the project solutions or publish any of these solutions to the web. Each of these projects relates to the following problem:

THE TANK TIME PROBLEM

Let's consider a system of three brine tanks containing V_1 , V_2 , and V_3 gallons of brine, respectively. A brine solution containing p pounds of salt per gallon flows into tank 1 at a rate of r gallons per minute, while mixed brine flows from tank 1 into tank 2, then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of r gallons per minute. Assume initially that tanks 1, 2 and 3 contain a , b , and c pounds of salt respectively. Let $x_i(t)$ represent the amount (in pounds) of salt in tank i at time t , for $i = 1, 2, 3$. We'll be interested in finding $x_i(t)$, of course. Let's see a graphic of this situation:



If we let: $\frac{r}{V_1} = \alpha$, $\frac{r}{V_2} = \beta$, $\frac{r}{V_3} = \delta$, and $kr = p$, then we obtain the system of three differential equations:

$$\begin{aligned} \frac{d}{dt}x_1(t) &= -\alpha x_1(t) && + p \\ \frac{d}{dt}x_2(t) &= \alpha x_1(t) - \beta x_2(t) \\ \frac{d}{dt}x_3(t) &= \beta x_2(t) - \delta x_3(t) \end{aligned}$$

A brief description of each of the three projects follows.

1. TANK TIME PROJECT I: We discuss some problems related to a cascade system of three brine tanks. This investigation leads immediately to a system of first-order linear equations. The theory of linear first-order systems and Maple provide us with a number of ways to solve these systems. One objective of this project is to show students how to apply some of these methods using Maple. We will actually use five different methods of solving the appropriate system of differential equations. The basic problem which

will consider can be found in the text *Differential Equations and Linear Algebra* by Edwards and Penney as Example 2 on page 493; however, we state the problem in the more general form. First we discuss various ways to use Maple to help us solve the basic problem as it appears in Edwards and Penney. Later, we try to answer some deeper questions about a more general form of the problem using Maple to help us. The focus of the latter part of this project is obtaining a complete answer to the following question: at any given time $t_0 > 0$ and any salt content c_0 what conditions on t_0 and c_0 are necessary to make it possible for all three tanks have content c_0 at time t_0 by appropriate choices for the initial salt contents a , b , and c of the three tanks?

2. TANK TIME PROJECT II: As in Tank Time Project I (see 1 above), we consider a cascade of three brine tanks. To be more specific, we consider the following problem: Consider a system of brine tanks containing V_1 , V_2 , and V_3 gallons of brine, respectively. A brine solution containing k pounds of salt per gallon flows into tank 1 at a rate of r gallons per minute, while mixed brine flows from tank 1 into tank 2, then from tank 2 into tank 3, and then out of tank 3, all at the same flow rate of r gallons per minute. Assume initially that tanks 1, 2 and 3 contain a , b , and c pounds of salt respectively. Let $x_i(t)$; represent the amount (in pounds) of salt in tank i at time t , for $i = 1, 2, 3$. Find $x_i(t)$, for $i = 1, 2, 3$. In this project, we fix the initial salt contents a , b , and c of the three tanks and the flow rate r . We allow k , V_1 , V_2 , and V_3 to vary and attempt to find conditions on these four variables which will give us 50 lb of salt in tank 1 after 1 minute, 100 lb of salt in tank 2 after 2 minutes and 150 lb of salt in tank 3 after 3 minutes. This project is completely independent of the project Tank Time Project I.

3. TANK TIME PROJECT III: In the solution of the problem described above in Tank Time Project II there are nine variables to contend with in our solution to the general problem: t , r , k , a , b , c , V_1 , V_2 , and V_3 . Clearly there are too many to deal with all at once! We will fix the values of seven of the nine variables and investigate the results one at a time. The variable t will never be fixed, so that means we will fix in turn seven of the variables r , k , a , b , c , V_1 , V_2 , and V_3 while keeping the eighth one variable. The "fixed" values we will be using are: $r = 10$ gallons/minute, $k = 2$ pounds/gallon, $a = 25$ pounds, $b = 50$ pounds, $c = 75$ pounds, $V_1 = 100$ gallons, $V_2 = 200$ gallons and $V_3 = 300$ gallons. We'll be interested in seeing the effect on our solutions of changing that eighth variable. Specifically, in each instance we will be interested in finding the values of those variables and the times t for which the salt contents of two or more tanks are equal. The amount of analysis required to complete this investigation is too much for a single project. So we will begin the investigation here but it will require two additional projects to complete it. These two additional projects have not yet been written! In this project we consider first the problem of varying the flow rate r . After this analysis, we consider the problem of varying the concentration of the incoming brine k . This project is completely independent of the projects Tank Time Projects I and II.

THE WEB SITE

The web site given below contains a sample of the differential equations Maple projects as well as some additional Maple worksheets. Please feel free to access this material and to use whatever you find appropriate in your own classes. The copyright on this material is, however, retained by myself.

<http://www2.SPSU.edu/math/fadyn/index.html>

The material can also be found by beginning at the Southern Polytechnic State University home page and then navigating to Professor Fadyn's home page. The home page for Southern Polytechnic State University is located at: <http://www.spsu.edu/>