The Impact of Dynamic Geometry Software as a Didactical Tool

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Abstract. In this note we illustrate via two examples basic capabilities of Dynamic Geometry Software (DGS) as a tool for teaching and learning geometry; in particular, the DGS potential to develop activities using the Inquiry-Based (I-B) approach and to promote discovery via the "explore-discover-conjecture-test-prove" model. In addition we comment on the impact of DGS both in our Concepts of Geometry as well as in the in-services we provide.

Dynamic geometry software (DGS) came to the market in the mid nineties. Since then, the use of this software as a tool for teaching and learning geometry in an interactive way has been growing at secondary and college levels in the USA and many other countries. As a user of the DGS Cabri, I have been increasingly impressed both by the capabilities and the pedagogical potential of this software. Thus, I prepared this talk with the following goals in mind:

1. To state some generalities about DGS for those non familiar with its use.
2. To describe how Cabri has impacted our Concepts in Geometry course and the training of pre-service and in-service teachers.
3. To illustrate with a couple of examples some of the capabilities that makes DGS an exceptional tool for teaching and learning geometry using the Inquiry-Based (I-B) approach and to promote discovery via the "explore-discover-conjecture-test-prove" model.

General observations

Cabri is easy to learn. The introduction that we provide to the students in our geometry course consists of grouping the students in pairs and having them complete the four step-by-step labs provided in the manual [1]. They do 1 or 2 labs the first day in a computer-equipped class under my supervision. After that they complete the remaining labs as homework for the next class. Thus, in one week most students are acquainted with enough Cabri tools to do independent work. Students learn additional tools as needed for their other labs.

Cabri does not require any algebraic background and provides continually visual feedback. One of my students comments that "Cabri reinforces what you are learning by your being able to easily verify to yourself that a property/result is true."

Cabri allows introducing, via discovery, geometrical concepts and properties ranging from elementary to advanced. The dynamic nature of this software makes experimentation and discovery inherent to its use from the very beginning. This in turn motivates the need for proofs as a way to establish the truth of generalizations found.

On how DGS has influenced our Concepts in Geometry course.

Our 4-credit course is taken mostly by Secondary Math students and some Math majors at the senior or graduate level. Initially our course was a traditional survey on geometry. Currently the course includes axiomatic Euclidean geometry, a review of relevant results in modern geometry, and a brief introduction to non-Euclidean geometries. The textbook used is College Geometry, by David C. Kay. Our guiding philosophy for teaching geometry is:

I. Geometry is better learned from the concrete to the abstract, hence should be as visual and hands-on as possible.
II. True learning takes place when students learn to raise questions, to explore and discover known principles, and dare to venture in new unknown territory.
III. The best way to answer a question and to convince ourselves and others of a result is a well-written convincing logical argument. Discovery is validated by proofs.

Using Cabri, students work in teams to solve a set of I-B activities on traditional, modern Euclidean and non-Euclidean geometries. Additional exploration and discovery are encouraged and rewarded in each activity. Students also do a final project that includes creating new IB activities, a poster session, and an in-class presentation.

Students use DGS (Cabri) to:

1. Review by exploring fundamental results from Euclidean geometry. This helps with the diverse, mostly weak, geometric background of many of our students.
2. Discover important results from modern Euclidean (9-pt circle, Sinson line,...) and non-Euclidean geometries.
3. Solve challenging problems involving lesser-known results.
4. Familiarize students with DGS as a tool to teach/learn by discovery important concepts from algebra and calculus.
5. Develop students' ability to raise their own questions and then try to solve them.

As a result, DGS contributes to:

i.) Increasing the depth and breadth of our course.
ii.) Making students more active participants in their learning process.
iii.) Aiding students to develop more confidence in their own ability to solve problems.
iv.) In many cases, initiating students to the research process.

Some examples to illustrate DGS capabilities

Problem 1. After the snowy winter of 2002-03, the mayors of Barberton, Cuyahoga Falls, and Stow gave a contract to the SS Company to build a common warehouse (W) to store salt for the snow trucks, and a straight road from the warehouse to each city. The cost of the warehouse was subsidized with federal funds.

i.) If the roads are also subsidized with federal funds, what will be the ideal –less troublesome– place to build the warehouse and how do we determine it?
ii.) If the company won the bid to build the warehouse with the condition that they must pay for the road connecting the location they choose with each of the three cities, what will be the more convenient place for the company to build the warehouse and how do they find the best location?

You may assume that the appropriate place to build the warehouse in either case is vacant!

Solution. Students easily identify that, since cost is not a factor, the best location should be equidistant from the three cities. Figure 1.1 displays the solution as the intersection of perpendicular bisectors or locus of points equidistant from two given points. The second question is far more difficult since it involves minimizing the sum of distances from the warehouse to the cities. Students use trial and error and some need a hint about looking at the angles formed.

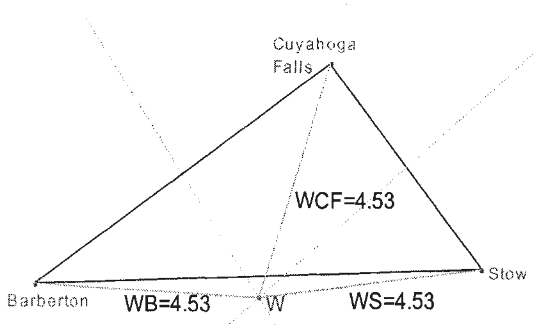

Figure 1.1

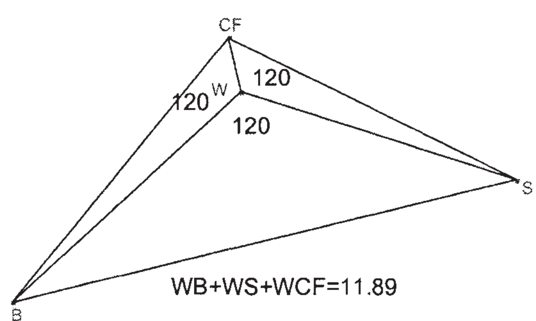

Figure 1.2

Problem 2. Your friend Joe has challenged you to find an object he buried in the park. He gives you two clues: i) the object is 50 feet away from the big rock, and ii) its distance from the water fountain plus its distance from the large oak tree is 120 feet. Studying the park you walk from the rock to the fountain and measure this distance to be 40 feet. Then you turn a quarter of a circle to your right and walk 90 feet to the oak tree. Can you find all possible points where the object might be buried?

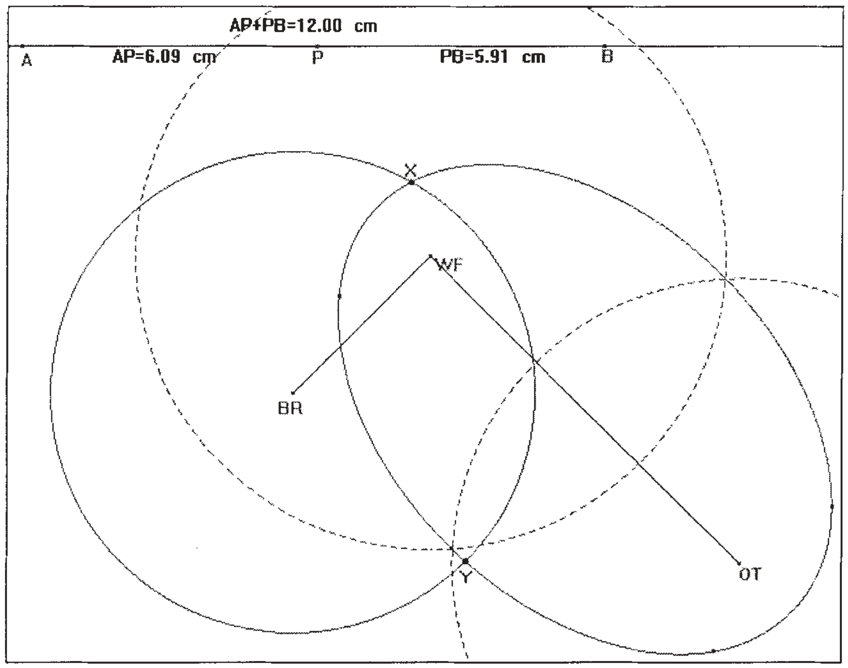

Figure 2.1

Solution. As seen in figure 2.1 the points X and Y are found as the intersection of the circle centered at BR of radius 50 with the ellipse with foci at WF and OT. The locus of each of the intersection points of circles of radius AP and PB centered at WF and OT provide the ellipse that can be redrawn using the tool conic.

Inquiry-Based (IB) Teaching and Learning Using DGS

Motivated by the results obtained in the geometry course, we decided to help spread the use of DGS to the schools by offering summer workshops for in-service training. Thanks to some Ohio Board of Regents grants, we were able to offer several in-services and to develop a model by which participant teachers first work as students with I-B activities, then they work in teams as teachers to develop new I-B activities that become part of a collection available to all that would be also used in the geometry course. A sample of a simple I-B activity is included in figure 3. We finish with a list of some of the lessons learned that may be of value for the reader:

1. Writing a good I-B activity is an excellent learning experience since it requires not only mastering the concepts involved but also thinking in the Socratic way.

2. Most secondary students have heard/learned about the I-B approach, but they have had very little experience, if any, learning with this type of materials, and no experience whatsoever developing I-B activities!
3. It is time consuming to develop good I-B activities!
4. Most students think of problems merely in terms of "book exercises;" few students have done problem solving outside that context.
5. When exposed to I-B for only one semester, some students (often a large %), mostly elementary teachers, do not value the I-B approach. They are not used to "exploring and/or discovering" so they lack confidence in their abilities and in their conclusions, and feel that this approach is a waste of time. In their view, "why discover a known result when it can be read and memorized?"
6. After completing an I-B activity, many students do not stop to review and grasp the concepts and properties discovered.
7. Initially, most students have difficulties developing IB activities. Sometimes teamwork is interpreted as a subdivision of goals w/o the appropriate planning and an ongoing exchange of ideas.
8. The I-B activities seem to have increased the self-confidence in solving real math problems of some students.

<u>Lab activity: **The Euler Line**</u>

1. Construct a triangle and label the vertices *A, B*, and *C*. [Use ***triangle*** tool.]
2. Construct the circumcenter of □ ABC and label it D. Hide the perpendicular bisectors.
   Construct the centroid of □ ABC and label it E. Hide the medians.
3. Construct the orthocenter of □ ABC and label it *H*. Hide the altitudes.
4. What do you notice about the relative positions of points *D, E*, and *F*? Test your conjecture using the appropriate test in Cabri under the icon "*?..*".
5. Construct a line through any 2 of the three centers to verify your answer.
6. Drag any of the vertices of □ ABC. What do you observe?
7. Define segments $\overline{DE}$, $\overline{EF}$ and $\overline{DH}$ and find their lengths DE = ___ EF = ____ DH = ____
8. Calculate the following ratios: *DE/EH* = _______ *DE/DH* = _______
9. Drag any of the vertices of □ ABC. What do you observe about the ratios *DE/EH* and *DE/DH*?

Make a chart including the segments and ratios for different positions of the vertex to verify your answer.

Position	*DE*	*EF*	*DH*	*DE/EH*	*DE/DH*
1					
2					
3					

10. What observations can you make about the "Euler Line" (the line containing *D, E*, and *H)* when the triangle is isosceles? Equilateral?

<u>Figure 3</u>

The space limitations of this note prevent us from showing examples of the use of DGS to teach relevant concepts from other areas of mathematics such as algebra and calculus.

Bibliography

1. Getting started with Cabri Geometry II for Mcintosh, Windows, and MS-Dos. Texas Instruments Incorporated, 1977.