

## Interactive Web-based Calculus Projects at Hollins University: Student Solutions

Julie M. Clark and Patricia W. Hammer  
Department of Mathematics and Statistics  
Hollins University  
Roanoke, Virginia  
[jclark@hollins.edu](mailto:jclark@hollins.edu) [phammer@hollins.edu](mailto:phammer@hollins.edu)

In the mathematics and statistics curriculum at Hollins University, there is much emphasis on the application of concepts to real life problems. For this purpose, we have recently developed two interactive web-based calculus projects which use JAVA applets as well as downloadable Maple worksheets. These projects are original and exciting and have been well received by both students and colleagues.

One Calculus II project was developed during the online PREP workshop hosted by Duke University in July 2002 (sponsored by the MAA and NSF) and was recently published in [The Journal of Online Mathematics and Its Applications](#). The project was written by Julie Clark, Trish Hammer, and Caren Diefenderfer of Hollins University, as well as Steve Hammer of Virginia Western Community College.

In this project, students use a “live” on-line map of Virginia to collect  $(x, y)$  pairs along the borders and then import these values into a downloadable Maple worksheet. This worksheet is complete with the commands (plot, solve, integrate, do loops) necessary for visual and mathematical construction of the area approximations. The project then teaches students to use four different approximation methods, progressing from rectangular and trapezoidal rules to the more complex Simpson’s rule, to estimate the area of Virginia. The southern boundary of Virginia is horizontal and so provides a nice “base” for the approximating figures. For more information about this part of the project, see [http://www1.hollins.edu/depts/math/Prep/area\\_of\\_virginia.htm](http://www1.hollins.edu/depts/math/Prep/area_of_virginia.htm).

As a final project assignment, students are required to extend the ideas developed in working with the state of Virginia to approximate the area of one of four carefully selected states, California, Illinois, Maryland, or North Carolina. These states were chosen because students cannot just mimic their work with Virginia. They must instead think carefully about how they should modify and extend the previous (Virginia) procedure. For example, the nice horizontal border in these states is the northern (rather than southern) border. Also, each of the four states has some "indentation" and irregularity in its shape.

Student solutions for this final project have been quite varied and creative. For the state of California, students begin by using the interactive map (Figure 1) to mark points along the boundary.



Figure 1

One student chose to sample two distinct sets of boundary points (red and blue) as shown in Figure 2 below. She then used Maple to calculate the area of the red trapezoids and the area of the blue trapezoids. She then subtracted the blue area from the red to arrive at a final approximation. Note that she chose to sample a great many points so as to increase her accuracy.

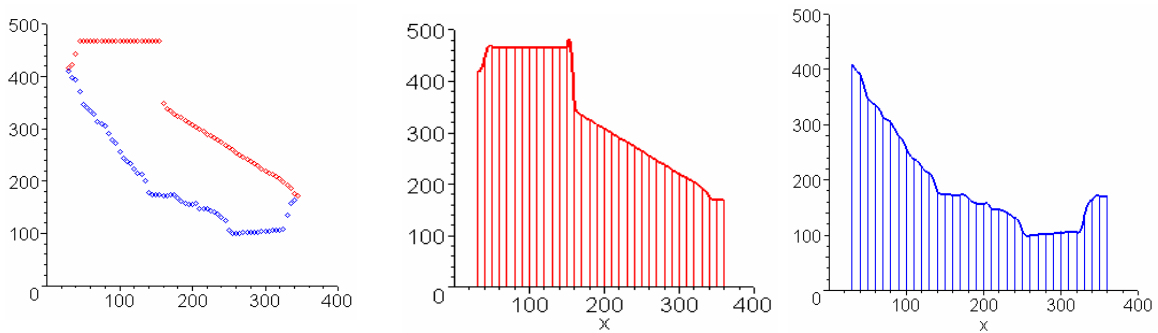


Figure 2

Another student chose a similar, but somewhat different approach. Rather than constructing trapezoids with horizontal bases along the  $x$ -axis, she turned her trapezoids “upside down” and used a line along the north edge of the state as the base of the horizontal trapezoids. (She must have enjoyed using the live-map to sample points because she sampled a very large number of them!) She then calculated blue trapezoid area minus red trapezoid area to approximate the area of California.

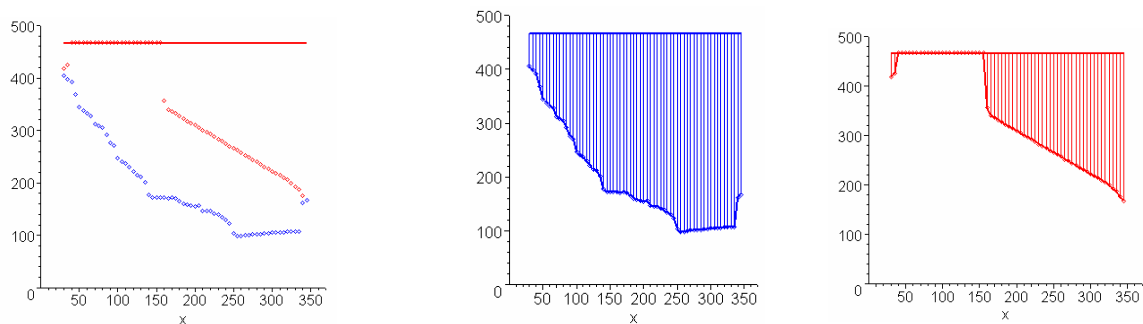


Figure 3

A third student took an entirely different approach to California (see Figure 4). She viewed the northern border of the state as a horizontal line and decided that the eastern edge could be considered a combination of a vertical and slanted line. Thus she sampled points only along the southwestern edges, used Maple to find the area of a large rectangle, and then subtracted from this rectangle's area the area of a (blue) trapezoid and the area of the red trapezoids shown below. Her approach meant she did not need to use the live-map to sample nearly as many coordinate pairs.

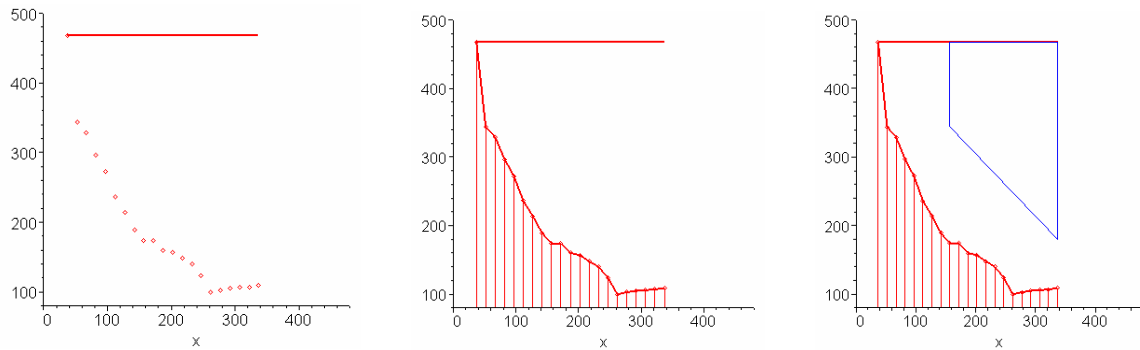
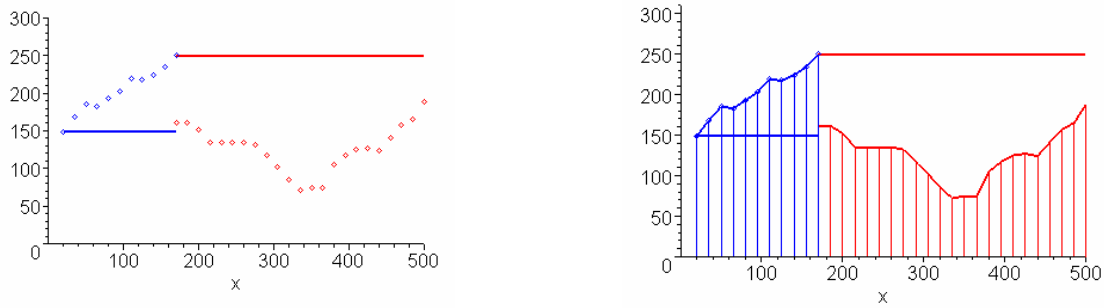


Figure 4

Figure 6 below shows how one student chose to estimate the area of North Carolina. She wanted to take advantage of both horizontal segments along the borders, so she used Simpson's Rule to estimate the area of the blue region and then subtract the area of the blue rectangular region. Then she found the area of the red rectangular region, used Simpson's rule to find the area of the red region below the state boundary, and subtracted this from the area of the red rectangle. Then she added the areas of the blue and red state regions.

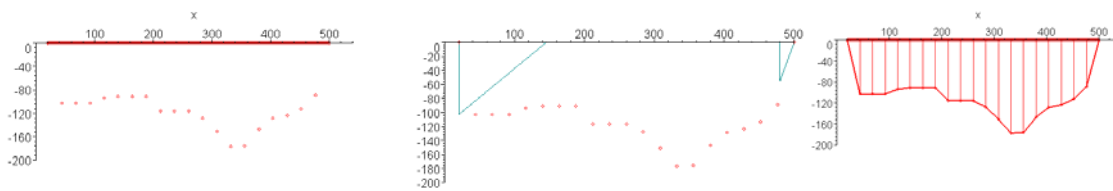


Figure 5



**Figure 6**

Another student viewed North Carolina a little differently (Figure 7). First she adjusted her  $(x,y)$  pairs so that the horizontal northern border of the state lined up with the  $x$  axis. She then sampled only along the southern border, determined area of these approximating trapezoids and subtracted easily computed areas of triangles.



**Figure 7**

Of course these assumptions about the triangular regions and her sampling of a relatively small number of points along the southern boundary resulted in an approximation that was not particularly accurate.

Figure 9 below demonstrates the approach one student took to finding the area of Maryland. The most interesting part of her approach was that she chose a state whose shape made the problem so difficult. Any of the other 3 choices would have been easier. She viewed the state as two separate regions, red and blue. For both regions, she found the area of the rectangular regions and then subtracted the area of the trapezoids.

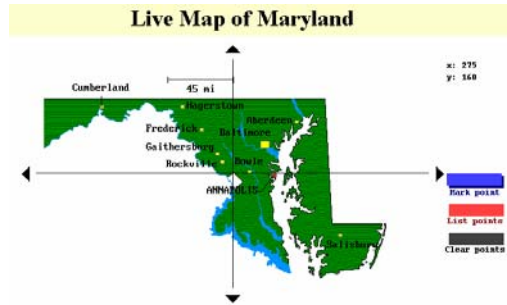


Figure 8

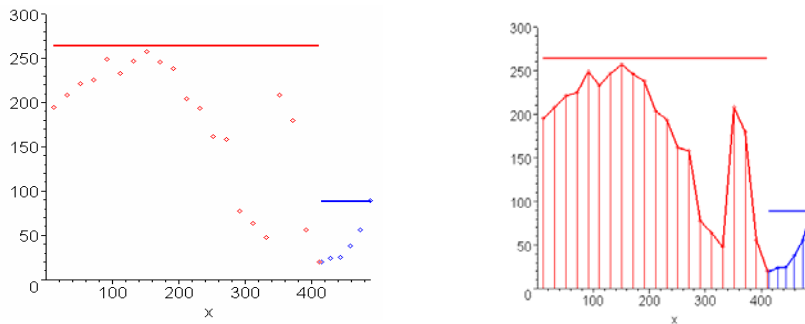


Figure 9

Surprisingly, no Hollins student has yet been willing to try estimating the area of Illinois (Figure 10).



Figure 10

In summary, we were very pleased with the student work on these projects. Students enjoyed the challenges and seemed particularly proud of their creative and colorful methods of solution. We believe their success was due to the organization of the project, where they were led carefully through the process for Virginia and through the initial Maple code, and then asked to think creatively about how to apply their ideas to other irregularly shaped states. With the availability of the interactive state maps and easy to modify Maple code, students found it easy (and fun!) to experiment with many different possible solutions and to see (almost immediately) their techniques in action. They were able to focus on the method of solution and not on Maple syntax. We have already created a similar interactive web-based Calculus I project on Design of a Thrilling Roller Coaster (<http://www1.hollins.edu/depts/math/hammer/coaster>) and have plans for several more in the near future.