Interactive Web-based Calculus Projects at Hollins University: Design of a Thrilling Roller Coaster

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In the mathematics and statistics curriculums at Hollins University, there is much emphasis on the application of concepts to real life problems. For this purpose, we have recently written two interactive web-based calculus projects which use JAVA applets as well as downloadable Maple worksheets. These projects are original and exciting and have been well received by both students and colleagues.

In the Calculus I project (www1.hollins.edu/depts/math/hammer/coaster/), students use differentiation concepts to mathematically model the path of roller coasters and to analyze the path of these coasters with regard to peak and valley points (local extreme points), points of steepest ascent and descent (inflection points), and "thrill." Students first apply these ideas to interactive on-line images of real coasters and then extend these ideas to their own coaster designs which must satisfy the following coaster restrictions:

- The total horizontal length of the coaster must be less than 200 feet.
- The track starts 75 feet above ground and ends at ground level.
- The track cannot exceed a height of 75 feet or go below ground level.
- No ascent or descent can be steeper than 80 degrees with the horizontal.
- The roller coaster must start and end with a zero degree incline.
- The thrill of a drop is the product of the radian measure of the angle of steepest descent with the vertical distance of the drop.
- The thrill of the coaster is the sum of the thrills of the drops.
- The path of the coaster must be modeled by a differentiable function.

Students use downloadable Maple worksheets to design and mathematically build the straight stretch coasters and to determine the corresponding thrill. Cubic polynomial functions and trigonometric functions are used to model the paths of these single drop and several drop coasters. As a project assignment, students are asked to design a coaster which satisfies the given restrictions and that has the maximum thrill. This optimization problem is a challenging one but the interactive coaster windows and the downloadable Maple worksheets make it easy (and fun!) for students to experiment with different designs. This project was developed by Professor Trish Hammer and student Jessica King from Hollins University and Professor Steve Hammer from Virginia Western Community College with support from the Hollins faculty/student research fund.

Students follow a step by step procedure which begins with a basic introduction to coaster design. In this introductory section, students gain an overall understanding of the methods being used. An interactive web window and Java applet allow students to simply point and click to mark peak and valley points. The window automatically

displays the cubic polynomial which fits their marked points along with the slope at the steepest points. Students then use this information to calculate the thrill of each drop and the total thrill for the coaster.

In this first section, students do not actually determine mathematical models. They simply see the mathematical models in action. Figure 1 below shows the automatically generated piecewise defined cubic polynomials that fit (in height and slope) the marked peak and valley point of one portion of the Steel Dragon. Students use the displayed slopes and y coordinates to calculate thrill.

angle1 = arctan(1.254) = 0.8976 radians = 51.4 degrees.

angle2 = arctan(1.0675) = 0.8180 radians = 46.9 degrees.

angle3 = arctan(1.34) = 0.9297 radians = 53.3 degrees.

angle4 = arctan(0.8579) = 0.7091 radians = 40.6 degrees.

Thrill = (217-115)*(0.8976)+(189-121)*(0.9297) = 154.7748

Figure 1
(coaster image from www.coastergallery.com)

Next, students begin to mathematically model the path of the coasters. Data in the form of peak and valley points is collected from the live on-line images of real life coasters and then students import these data points into a downloadable Maple worksheet. The Maple worksheet is complete with the commands (plot, solve, differentiate, do loops) necessary for visualization and mathematical computation and analysis of the mathematical models. We choose to use actual worksheets rather than Maplets because students at Hollins are expected to learn to read, write and understand Maple code.

Four modeling methods are presented. Students begin with modeling just a single coaster hill and then progress to modeling a roller coaster with several hills. In each case, students are asked to use cubic polynomials and then trigonometric functions. For each method, students are given specific step by step instructions that lead them through the mathematical modeling process. At the end of each of the four sections, students are asked to model their own coaster path by first creating an interesting design and then by specifying peak and valley points on the path. Their goal is to design a coaster path which obeys the given restrictions (regarding height, slope, and differentiability) and which is also thrilling.

For example, to mathematically model several drops of the Steel Dragon coaster, students enter the collected x- and y-coordinates of the peak and valley points (marked in Figure 1) into a downloadable Maple worksheet. They then use Maple commands to fit a cubic

polynomial (or a trigonometric function) to each consecutive pair of points by creating a system of four equations in four unknowns. Two equations result from the matching of *y*-coordinates and two equations result from the matching of slopes (first derivatives.) With Maple, students are able to quickly graph their piecewise mathematical model to see if it does in fact match the real coaster image. Students then use first and second derivatives to identify the points of steepest ascent and descent (points of inflection) and the corresponding slope at these points. The slopes are then converted to corresponding angles (with the horizontal) and students compute the thrill of each drop of the coaster, and finally the total thrill of the coaster. A student generated mathematical model for the Steel Dragon path marked in Figure 1 is shown below in Figure 2. The degree measures of the angles of steepest descent and ascent are also displayed. "SAFE" indicates that no ascent or descent is steeper than 80 degrees with the horizontal.
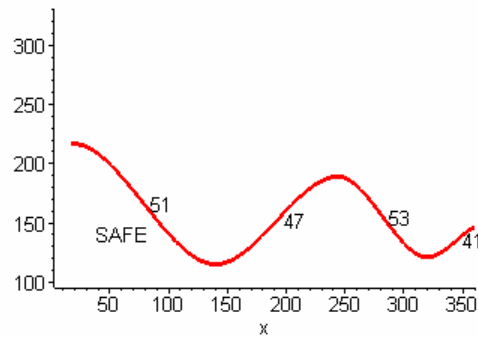
Figure 2

In the last section of the project, students must complete a project assignment. In this assignment, students are asked to create a coaster that satisfies all coaster restrictions and that is the most thrilling. This optimization project is a challenging one. How do they create the "most thrilling" coaster? Should they use many shorter hills or fewer taller hills? The solution to this optimization problem requires the use of non-peak/valley points and non-zero slope conditions, so students must think carefully about how to modify their approach and the Maple code. For this project assignment, students must submit mini-technical reports that contain a complete mathematical explanation of their optimal design along with the Maple worksheets.

A preliminary version of this project has been used for many years at Hollins University and at Virginia Western Community College. In this version, students were shown an example of a mathematical model (using Maple) for a roller coaster and then asked to model a roller coaster that was most thrilling. Students who were "good" at Maple found the project exciting and were able to experiment easily to find the best coaster design. Those students who were not as "good" at Maple struggled, found the project (and Maple syntax) frustrating and were just happy to produce one mathematical model. We believe the addition of the interactive images that allow point and click experimentation along with easily modifiable Maple code will allow all students a quick and easy way to check

out their "hunches." They will then be able concentrate on mathematical concepts and not on Maple syntax.