MATHSKILLS

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" *In the company of friends, writers can discuss their books, economists the state of the economy, lawyers their latest cases, and businessmen their latest acquisitions, but mathematicians cannot discuss their mathematics at all*." *Alfred Adler*

Introduction

Communicating mathematics to different audiences should be an integral part of any modern mathematical degree, since without this fundamental mathskill we will become (and are becoming) an endangered species. It is of great comfort to us and our fellow mathematicians that we can converse in a totally alien (to most people) language of say algebra and prove formally many different theorems. However a quarter of 16 to 65-year-olds in the world's richest countries are functionally illiterate. This means they cannot understand brochures, train timetables, road maps, and simple instructions for household appliances.(http://www.literacytrust.org.uk/database/internat.html#infoage) Viewing the whole world the this figure rises to 60%. So we must ask the question what percentage of the world can we communicate our algebraic forms to? - perhaps 0.1% if we are optimistic!

This paper presents some of our exciting and current experiences in the evolution of our mathematics degree at Sheffield Hallam University to address the issues and needs of mathematical skills now and for the future. The practices that we have adopted in Mathematics at Sheffield Hallam University- a large University by UK standards, to try to install range of modern mathematical skills (mathskills) in a technological rich context, aiming to make students reflect and engage in their learning.

What skills for mathematicians?

The students looked at here are on a UK undergraduate mathematics degree, and they study courses a rich diet of usual mathematical courses. Students on the degree carry out critical study and develop the full range of mathskills, both mathematical and general, in parallel. Student engagement in the work is steered by the use of an electronic web-based planning and progress file in which they can also reflect on their learning, such reflection being an important part of improving their skills. The thinking here is that there is a set of traditional *key* skills: Communication skills; Improving own learning and performance; Using Information Technology; Problem-solving; Application of number; Working with others should all be fully integrated into modern mathematics courses. The importance of these key skills is widely mentioned by employers [1] and a recent report [2] reinforced this view. Mathematics is essentially a communication language, it is critical that anyone with knowledge can recognize the audience with whom they are communicating. Without this last skill the most knowledgeable practitioner will not succeed. Here we examine a particular problem and relate it to the mathskills.

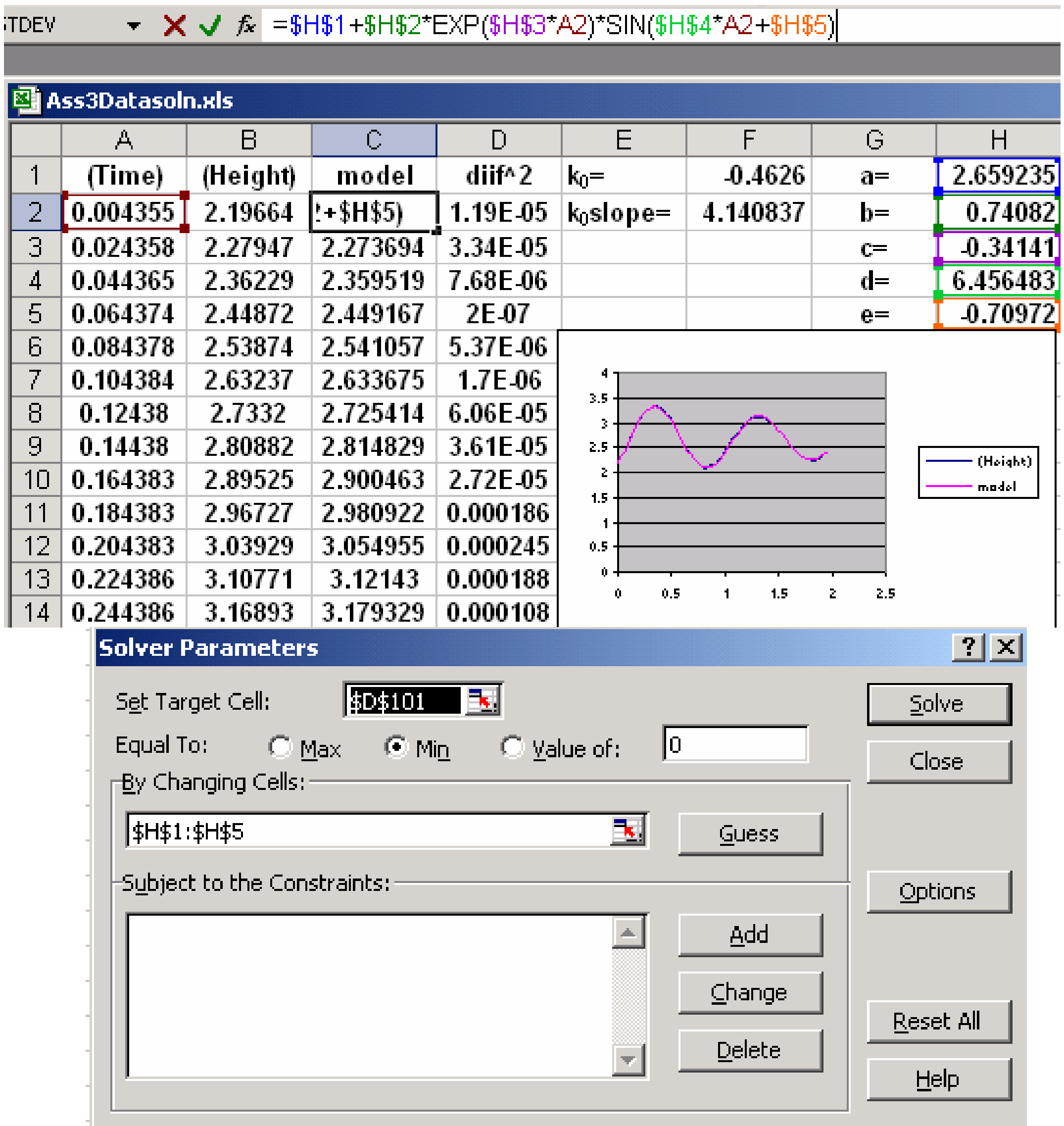
The problem - The bungee jumping car - damped *real* vibration

First we collect data from the motion of an oscillating model car – see Figure 1.

Figure 1

This starts the important process of engagement and ownership of the problem. The students are then given their tasks – initially they work in groups and collect their own data of height against time for the car on the end of elastic. A sample of the data (which was collected using a CBL2 , Motion detector and a TI-83p) is available at ftp://maths.sci.shu.ac.uk/hwg/DataBooks/EAZ6.doc

Note how the IT skills are embedded in the problem. The students are then asked to "Analyse and explain in your own words" a model to describe what is happening and suggested that the formula $y = A + B\, e^{(C*x)} \sin(D * x + E)$ could a useful start .But how to find the parameters? The students had earlier encountered the Excel solver to implement the least squares method so can use their experience to find the parameters for the model. Figure 2 gives some screen shots of the results.

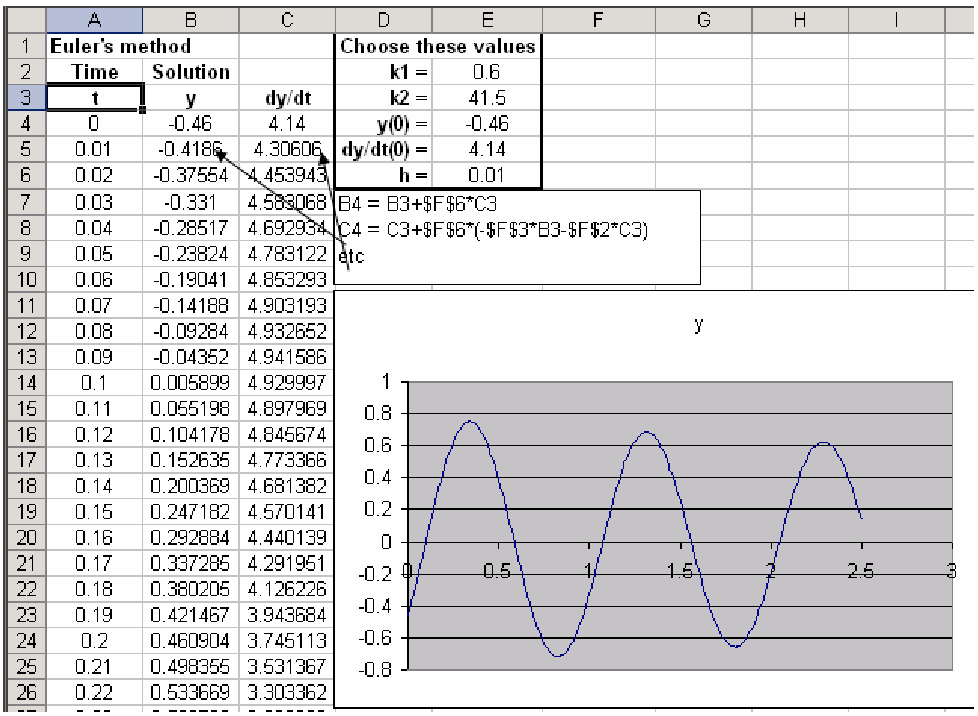


	A	B	C	D	E	F	G	H
1	(Time)	(Height)	model	diif^2	k_0=	-0.4626	a=	2.659235
2	0.004355	2.19664	!+$H$5)	1.19E-05	$k_0$slope=	4.140837	b=	0.74082
3	0.024358	2.27947	2.273694	3.34E-05			c=	-0.34141
4	0.044365	2.36229	2.359519	7.68E-06			d=	6.456483
5	0.064374	2.44872	2.449167	2E-07			e=	-0.70972
6	0.084378	2.53874	2.541057	5.37E-06				
7	0.104384	2.63237	2.633675	1.7E-06				
8	0.12438	2.7332	2.725414	6.06E-05				
9	0.14438	2.80882	2.814829	3.61E-05				
10	0.164383	2.89525	2.900463	2.72E-05				
11	0.184383	2.96727	2.980922	0.000186				
12	0.204383	3.03929	3.054955	0.000245				
13	0.224386	3.10771	3.12143	0.000188				
14	0.244386	3.16893	3.179329	0.000108				

Figure 2

Note the cell $D$101 = SUM(D2:D100) is minimised by changing cells $H$1 : $H$5 and a good guess is needed- estimation of solution is a necessary skill needed to solve real problems. The visualisation of the model solution and the collected data is a very powerful expressive and clear confirmation that the process is correct!
The initial conditions needed for the theoretical second part of the question k_0 =B2-H1 and $k_0$slope =(B3-B2)/(A3-A2) are shown in the spreadsheet as the second part of the problem starts. In the second part we are trying to raise theory to the level of practice, by relating the reality to the standard theory. The students download from the web a simple Excel spreadsheet template which implements the simple Euler's method for solving the differential equation $\frac{d^2y}{dt^2} + k_1 \frac{dy}{dt} + k_2 y = 0$. They are then asked to identify values for k_1, k_2 and the initial conditions to make the solution of this equation correspond to the data

collected and provide evidence that it works and fits. Some hints are often needed, for instance, when matching the two parts it helps to treat A as zero. The questions can be asked as to whether this is reasonable or not? together with what does A signify?) The students then have to experiment with the Euler Sheet template and find the theoretical values of k_1 and k_2.
Figure 3 shows the result.



	A	B	C	D	E
1	Euler's method			Choose these values	
2	Time	Solution		k1 =	0.6
3	t	y	dy/dt	k2 =	41.5
4	0	-0.46	4.14	y(0) =	-0.46
5	0.01	-0.4186	4.30606	dy/dt(0) =	4.14
6	0.02	-0.37554	4.453943	h =	0.01
7	0.03	-0.331	4.583068	B4 = B3+$F$6*C3	
8	0.04	-0.28517	4.692934	C4 = C3+$F$6*(-$F$3*B3-$F$2*C3)	
9	0.05	-0.23824	4.783122	etc	
10	0.06	-0.19041	4.853293		
11	0.07	-0.14188	4.903193		
12	0.08	-0.09284	4.932652		
13	0.09	-0.04352	4.941586		
14	0.1	0.005899	4.929997		
15	0.11	0.055198	4.897969		
16	0.12	0.104178	4.845674		
17	0.13	0.152635	4.773366		
18	0.14	0.200369	4.681382		
19	0.15	0.247182	4.570141		
20	0.16	0.292884	4.440139		
21	0.17	0.337285	4.291951		
22	0.18	0.380205	4.126226		
23	0.19	0.421467	3.943684		
24	0.2	0.460904	3.745113		
25	0.21	0.498355	3.531367		
26	0.22	0.533669	3.303362		

Figure 3

Again Solver helps here and yet again a good guess is essential. This guesswork only comes with the understanding of the real situation and enables the values of k_1 and k_2 to be clearly interpreted. Finally the student has to reflect, comment and write up their findings often to different prescribed audiences. Our formal course document states *"Skills work will include input on aspects of report writing including written English and referencing, information retrieval, curriculum vitae, group working, time management, reflection and action planning."* Finally their reports are put on the students own web page for *theirs* and *our* future reference[3].This particular problem has provided a valuable exercise in mathskills which together with the identification of audience provides an excellent ending. The assessment of these skills has been a learning process for staff involved as well and is documented in [4] and [5]. This process is a prime necessity in a world now mainly composed of strategic learning by most students.

Discussion

What we are doing here is using an approach to learning mathematical skills which is unusual, although perhaps less so in the arts and humanities. Degrees in other disciplines make widely different demands. Mathematics has been predominantly by theory and algebra whereas other areas have been more discursive based, with correspondingly better perceived attainment levels. We are to some extent trying to readdresses this balance. This is necessary as the mathskills needed now have changed due to technology and the different needs of society and we need a consistent way of assessing the full range of mathskills much valued by employers.

Conclusion

Here a simple real problem has been documented to show how mathskills can be embedded into a “mathematics” course. Mathematics students initially are very loathe to write words and use pictures (since they are the chosen few who know and are comfortable with algebra!) but they have to realize that most problems will arise in these formats. Here we are hoping to encourage an attitude of reflective, continuous engagement, appropriate for a technology-intensive and changing world. At the end of all our courses the students have an electronic web based portfolio covering all their modules, providing evidence of all their work and expertise both mathskills and general. This provides a much fuller picture of their abilities than would just final marks.This kind of profile is something that employers considering employing our graduates welcome. Learning is never finished both for us and the students!

References

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[3] Waldock J ,Gretton H.W. and Challis N.V. (2002), ”Using the Web to Enhance Student Learning”, *Proc 2nd International Conference on the Teaching of Mathematics*, University of Crete, Hersonissos, <http://www.math.uoc.gr/~ictm2/Proceedings/pap199.pdf> (accessed August 28th 2003)

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[5] Waldock J ,Gretton H.W. and Challis N.V. (2002), **“**Assessing Mathskills using the web 100 marks is not enough!” *Proc. 15th International Conference on Technology in Collegiate Mathematics*, Addison Wesley Longman (too appear)