Using Excel Spreadsheet to Understand the Limiting Value of a Sequence

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ICTCM 2003 November 2, 2003
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Abstract

Problems involving sequences and series are rarely common in science and mathematics subjects. Especially for students to find the limiting value of a sequence, in other words to determine the convergence or divergence of a sequence, is still a confusing part. This confusion can be solved numerically and graphically using the application of Excel spreadsheet techniques. Students could visualize the limiting value of a sequence. Students' understanding about the convergence of a sequence was overwhelming.

Introduction

In this paper I discuss how the Excel spreadsheet can be used to understand well about the convergence of a sequence and the behavior of functions where independent variable approaches to infinity (∞). I chose three most interesting sequence problems which are always intricate for students to see the correct answer analytically.

The problems discussed are: :

(i) $Limit_{n \to \infty} n^{\frac{1}{n}} = 1$

(ii) $Limit_{n \to \infty} (1 + \frac{1}{n})^n = e$

(iii) $Limit_{n \to \infty} (\frac{\sin n}{n}) = 0$

Problem No. 1 ➜ $Limit_{n \to \infty} n^{1/n} = 1$

Finding the limit of function $n^{1/n}$ when n approaches to infinity (∞) is analytically very confused for freshmen level students. The complex part for them is to get the concept of an infinite number in the real world. This complex concept of infinity can be clearly explained by Excel techniques as shown in figure 1, and figure 2.
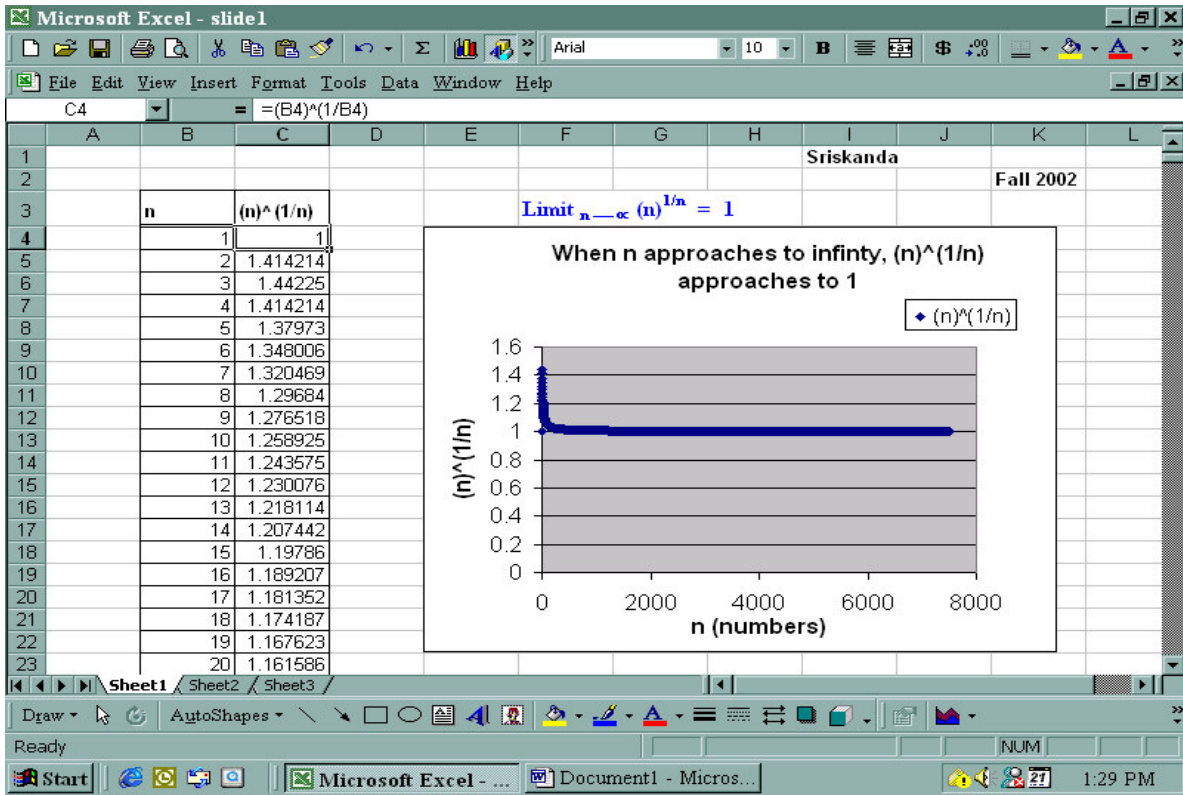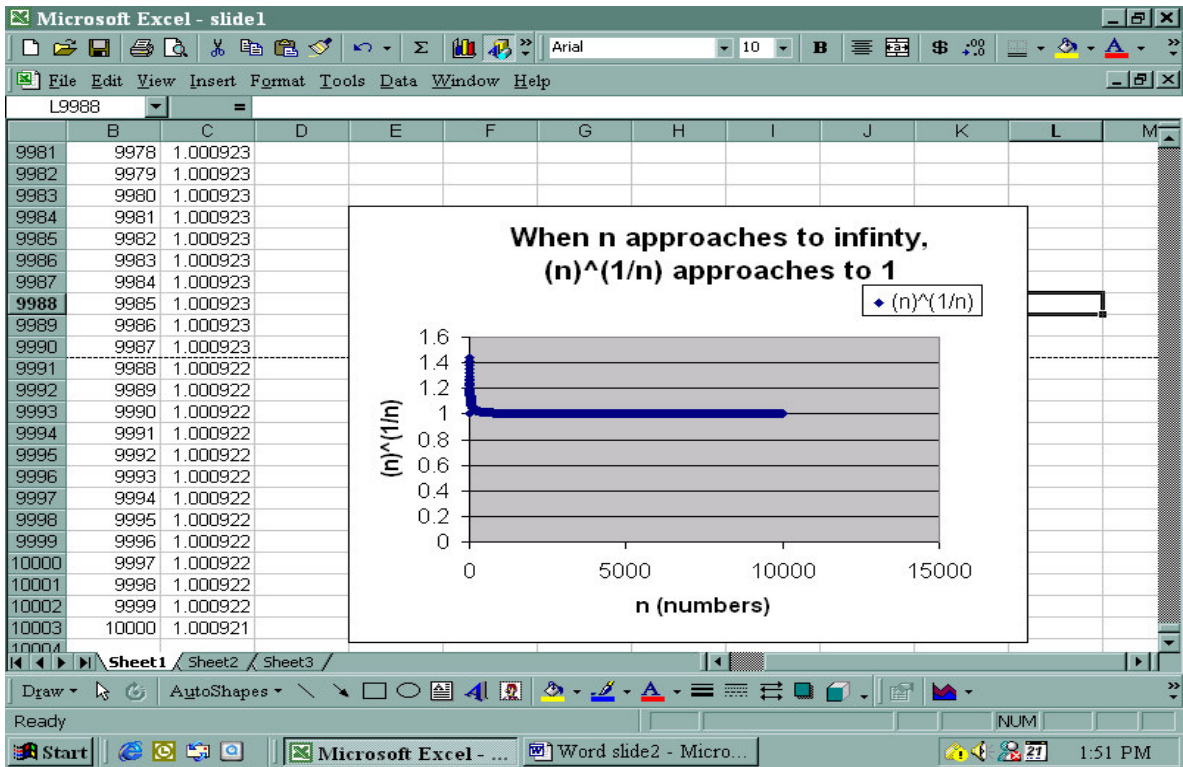
Figure 1

Figure 2

One can see in figure 1 that when n =1 the function get the value of 1 and the function gets the maximum value of 1.44225 when n= 3. Then the function value decreases as n gets larger. In figure 2, n reaches to a bigger number of 10000 and the function value is equal to 1.000922 which is equal to one (1) for first three decimal places. This can be explained to students that n gets bigger and bigger the function value gets closer and closer to one (1) for more accurate significant numbers. As time permits, students can go up to 65000 iterations (n➜65000) in Excel, which is closed enough to assume theoretically that n approaches an infinite number.

Problem No. 2 ➜ $Limit_{n\to\infty}(1+\frac{1}{n})^n = e$ where e = 2.7052…, an irrational number, is still a difficult part to explain the limiting process of the nth term sequence analytically to an undergraduate level math major student. In calculus we use logarithm and the L'Hopital's
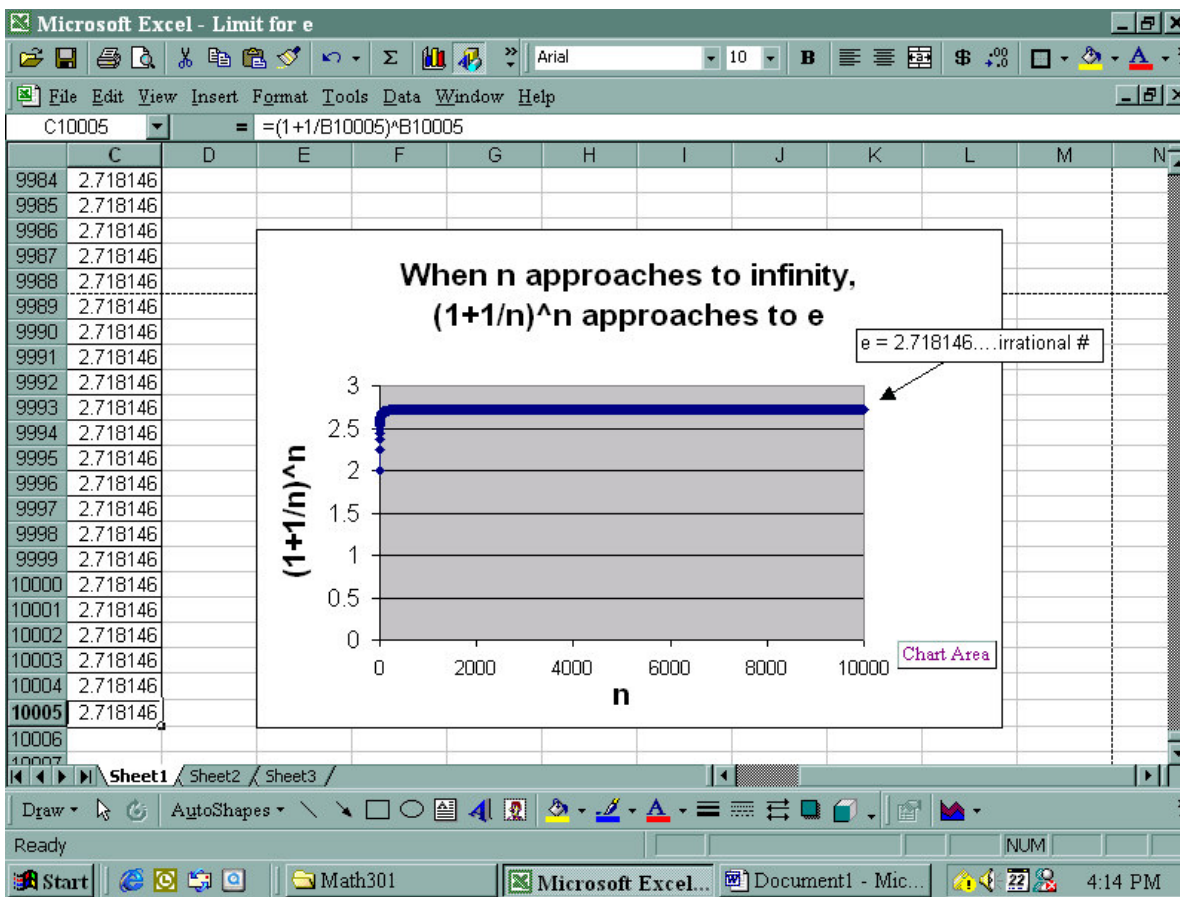

Figure 3

rule to analyze the limiting value of this sequence. Using Excel spreadsheet one can create a table for larger n values, in this case I go up to the nth value of 20,000 and notice that for nth values between 10,000 and 20,000 the function value for $(1+1/n)^n$ is equal to a constant number, 2.71821, for same five decimal places. This makes an assumption that when n goes to infinite number the function $(1+1/n)^n$ approaches to a constant value of e ($\approx 2.21821...$).

Problem No. 3 ➔ $Limit_{x\to\infty} \frac{\sin x}{x}$

The proof of $Limit_{x\to\infty} \frac{\sin x}{x} = 0$ in applied calculus course is demonstrated using the Squeeze Theorem in which $\frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$ where $Limit_{x\to\infty} \frac{1}{x} = Limit_{x\to\infty} \frac{-1}{x} = 0$. In figure 4, I
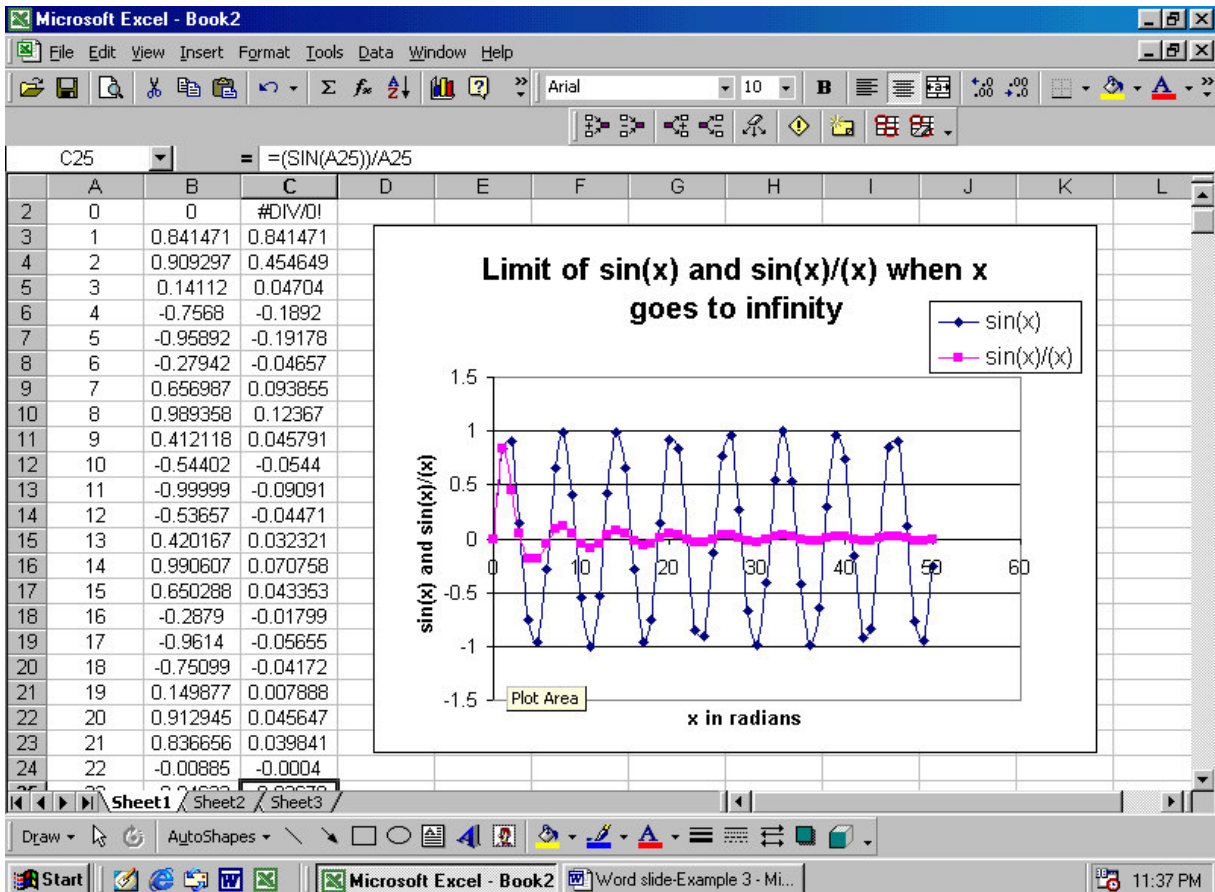

Figure 4

used Excel to create a table for the functions sin(x) and $\frac{\sin x}{x}$ and also incorporate the graphs which may assist the students to view the behavior of the functions when the x-value goes to a larger number (∞). Students can see in the graph that the sine function varies nicely between the values of -1 and 1. This is a well known fact in trigonometry. But the function value of $\frac{\sin x}{x}$ when x approaches to infinity is not easily to be explained for many calculus students. Here by doing in Excel spreadsheet as shown in figure 3 students can view the waves of the graph of $\frac{\sin x}{x}$ converges smoothly for a constant value of 0.

Conclusion

Microsoft Excel spreadsheet is a very common software in every household and in every educational institution in the United States. The students particularly in mathematics can get the concept of approachness to infinity using the rows in Excel as an independent variable where number of rows in Excel is almost 64000. Students' understanding about the infinity was overwhelming.

References

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