

# Potential Pitfalls for Students Using Online Math Utilities

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In early 2003, Wright State University installed several computers in its Mathematics Learning Center in order to provide access to mathematical applications on the Internet. Since then, students enrolled in introductory courses and the student-employees who staff the center have explored the capabilities of online math utilities at a variety of Web sites. Collectively, these utilities (called calculators or solvers at some sites) have delivered many of the features of a computer algebra system through the familiar interface of a browser. The utilities have been helpful in reinforcing concepts, generating exercises, and confirming solutions, but they have limitations that may undermine the unsuspecting student. Five of these weaknesses are described in this paper.

**A utility may perform well in some but not all cases of a type of problem.** Compare the output obtained at two sites for the following linear equations, which have one, zero, and infinitely many solutions, respectively.

$$3x - 17 + 5 = 6x - 4 - x$$

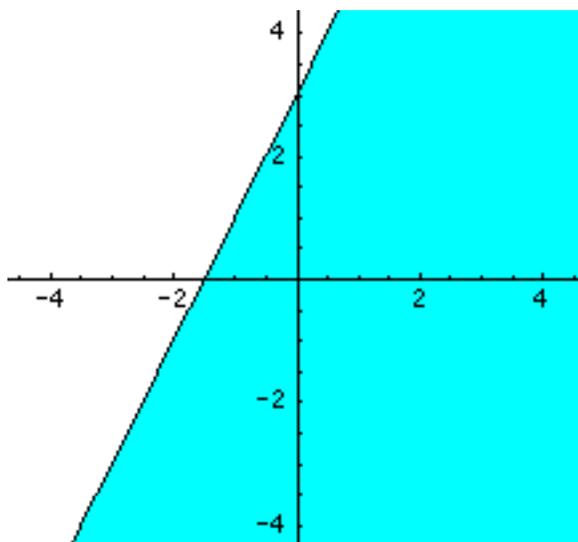
$$3x - 17 + 5 = 2x - 10 + x$$

$$3x - 17 + 5 = 2x - 12 + x$$

While algebrahelp.com<sup>1</sup> handles the trichotomy nicely, freemathhelp.com<sup>2</sup> gives the same empty result for the two extreme cases (complete URLs are listed in the endnotes).

	$3x - 17 + 5 = 6x - 4 - x$	$3x - 17 + 5 = 2x - 10 + x$	$3x - 17 + 5 = 2x - 12 + x$
algebrahelp.com	$x = -4$	There is no solution.	All real numbers are solutions.
freemathhelp.com	$x = -4$	(no output)	(no output)

Utilities may not respect subtle yet significant differences between cases. The Plot Inequalities utility at math.com<sup>3</sup>, for example, creates an adequate graph of  $y \leq 2x + 3$  (below) but then generates the same graph for  $y < 2x + 3$  instead of distinguishing the boundary line for the strict inequality.



**A utility may return a correct answer but not the intended one.** Consider the following integral:

$$\int x(3x^2 + 1)^7 dx$$

Integration by substitution with  $u = 3x^2 + 1$  yields a compact answer.

$$\int x(3x^2 + 1)^7 dx = \frac{1}{6} \int u^7 du = \frac{1}{6} \left( \frac{1}{8} u^8 + C \right) = \frac{1}{48} u^8 + C = \frac{1}{48} (3x^2 + 1)^8 + C$$

The integration utility at wolfram.com<sup>4</sup> provides this cumbersome answer:

$$\frac{x^2}{2} + \frac{21x^4}{4} + \frac{63x^6}{2} + \frac{945x^8}{8} + \frac{567x^{10}}{2} + \frac{1701x^{12}}{4} + \frac{729x^{14}}{2} + \frac{2187x^{16}}{16} \text{ (eight terms but no } C!),$$

which results from expanding  $x(3x^2 + 1)^7$  and integrating term by term.

Utility results that initially confound students may nevertheless lead to splendid teaching opportunities. In the previous example, for instance, what guidance could be given to a student who must compare an answer of  $\frac{1}{48}(3x^2 + 1)^8 + C$  to the long one generated by wolfram.com? Can the student identify the number of terms in  $\frac{1}{48}(3x^2 + 1)^8 + C \dots$  or perhaps the coefficient of the term with the largest exponent? These informal checks begin to build the case that each expression could plausibly be rewritten as the other. Of course, the savvy student would use another utility to expand  $\frac{1}{48}(3x^2 + 1)^8$ !

It is not clear whether the Simplify utility at quickmath.com<sup>5</sup> acts on  $\frac{x(x+3)-18}{x^2+2x-3}$  because

the “problem” is echoed as the “result.” Given that the objective of the textbook section in which this problem appears is to cancel common factors in the numerator and denominator, students

wonder, quite understandably, whether the utility has checked for such factors. Both  $\frac{x^2 + 3x - 18}{x^2 + 2x - 3}$

and, even better,  $\frac{(x+6)(x-3)}{(x+3)(x-1)}$  are acceptable answers that would have indicated processing by the

utility.

**A utility may provide approximate answers only.** According to math.com<sup>6</sup>, the solutions of  $2x^2 + 5x - 50 = 0$  are 3.903882032022076 and  $-6.403882032022076$ . In spite of all these digits, the answers are not as useful in debugging written work as the exact answers:

$$\frac{-5 + 5\sqrt{17}}{4} \text{ and } \frac{-5 - 5\sqrt{17}}{4}.$$

Surprisingly, another utility at the same site<sup>7</sup> provides both the exact and approximate solutions (below left). An amusing side effect of this utility is that an equation with integral solutions, such as  $x^2 - 2x - 8 = 0$ , gets the treatment shown on the right.

$$2x^2 + 5x - 50 = 0$$

$$x^2 - 2x - 8 = 0$$

## Result

### Exact

**Solution 1** (real)

$$x = \frac{5}{4} \left( -1 - \sqrt{17} \right)$$

**Solution 2** (real)

$$x = \frac{5}{4} \left( -1 + \sqrt{17} \right)$$

### Approximate

**Solution 1** (real)

$$x = -6.40388$$

**Solution 2** (real)

$$x = 3.90388$$

## Result

### Exact

**Solution 1** (real)

$$x = -2$$

**Solution 2** (real)

$$x = 4$$

### Approximate

**Solution 1** (real)

$$x = -2.$$

**Solution 2** (real)

$$x = 4.$$

**A utility may allow the user to customize a problem with impossible conditions.**

For example, one of the initial age problems at algebra.com<sup>8</sup> can be modified to the following:

Bob's father is  times (as) old as Bob.  years ago, he was  times older. How old is Bob?

The user is prompted to choose a quantity to be represented by  $x$  and to identify an appropriate equation. The utility mechanically solves the equation and reports that "Bob's age is  $-30$ ."

When the user chooses values to complete the statement of a problem, the Web site should check that the values represent a legitimate problem before proceeding. This approach is used at gomath.com<sup>9</sup>: when 3, 4, and 10 are entered as lengths of the sides of a triangle, the utility responds with a message that "the sum of any two sides must be greater than the third side, please try again."

**A utility may give poor, even meaningless, directions for its use, creating a logical disconnect between input and output.** For instance, what should users of the Sine Calculator at onlineconverters.com<sup>10</sup> enter for the oxymoronic "hypotenuse angle?" Experimenting with a few values reveals that the utility is dividing the number in the first box (not an angle measure but the *length* of the opposite *side*) by the number in the middle box (the length of another side, the hypotenuse).

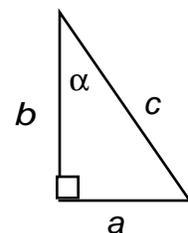
Enter the opposite and hypotenuse angle values and JavaScript will solve the sine angle for you.

Opposite	Angle	Hypotenuse	Angle	Sine Angle
<input type="text"/>		<input type="text"/>		<input type="text"/>
<input type="button" value="Calculate!"/>				

The redesign below would better indicate what the utility does:

Enter the lengths of the opposite side  $a$  and the hypotenuse  $c$  and the Sine Calculator will find the sine of the associated angle  $\alpha$ .

length of side $a$	length of hypotenuse $c$	$\sin \alpha$
<input type="text"/>	<input type="text"/>	<input type="text"/>
<input type="button" value="Calculate!"/>		



The discovery.com<sup>11</sup> utility to factor trinomials suffers from a poorly labeled button (“Solve!” should be “Factor!”) and unpredictable performance. When  $x^2 - 4x - 9$  and  $x^2 - 4x - 10$ , two trinomials not factorable over the integers, are submitted, the utility reports that  $x^2 - 4x - 9$  “cannot be factored as the product of two binomials” (indeed, a remainder is given beforehand that such an outcome is possible) but that  $x^2 - 4x - 10$  *can* be factored, incongruously, as  $(x - 5.74166)(x + 1.74166)$ .

In summary, online math utilities sometimes generate incomplete, incorrect, or ambiguous results. Students and instructors must recognize that impressive sites may have limitations that reduce their educational value. The powerful utility that supports the teaching and learning of mathematics may also be a problematic one.

### Endnotes (verified October 2003)

- <sup>1</sup> [www.algebrahelp.com/calculators/equation/solving/pg2.htm](http://www.algebrahelp.com/calculators/equation/solving/pg2.htm)
- <sup>2</sup> [www.freemathhelp.com/calcsolveequation.html](http://www.freemathhelp.com/calcsolveequation.html)
- <sup>3</sup> [www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=mathcom&s1=graphs&s2=inequalities&s3=basic](http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=mathcom&s1=graphs&s2=inequalities&s3=basic)
- <sup>4</sup> [integrals.wolfram.com/index.en.cgi](http://integrals.wolfram.com/index.en.cgi)
- <sup>5</sup> [www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=quickmath&s1=algebra&s2=simplify&s3=basic](http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=quickmath&s1=algebra&s2=simplify&s3=basic)
- <sup>6</sup> [www.math.com/students/calculators/source/quadratic.htm](http://www.math.com/students/calculators/source/quadratic.htm)
- <sup>7</sup> [www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=mathcom&s1=equations&s2=solve&s3=basic](http://www.hostsrv.com/webmab/app1/MSP/quickmath/02/pageGenerate?site=mathcom&s1=equations&s2=solve&s3=basic)
- <sup>8</sup> [www.algebra.com/cgi-bin/word-problem.mpl?problem\\_id=age/02-age.wpm&action=customize](http://www.algebra.com/cgi-bin/word-problem.mpl?problem_id=age/02-age.wpm&action=customize)
- <sup>9</sup> [www.gomath.com/algebra/triangles.php](http://www.gomath.com/algebra/triangles.php)
- <sup>10</sup> [www.onlineconverters.com/sine.html](http://www.onlineconverters.com/sine.html)
- <sup>11</sup> [school.discovery.com/homeworkhelp/webmath/factortri.html](http://school.discovery.com/homeworkhelp/webmath/factortri.html)

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