REMODELING CLASSIC CALCULUS CONCEPTS

Judy O'Neal
North Georgia College & State University
Department of Mathematics & Computer Science
Dahlonega, GA 30597
joneal@ngcsu.edu

Engaging students in learning traditional calculus concepts using a non-traditional format has become much easier due to recent technological advances. A digital camera, *The Geometer's Sketchpad (GSP)* software, and *TI-InterActive*! software are valuable tools for capturing models appropriate for determining areas and volumes of solids of revolution found in everyday contexts. The availability and affordability of digital cameras has eased the difficulty of including real-world objects as a motivator and means of developing students' understanding of how area and volume of such objects are computed. Coupling this with the inclusion of images that students find on the web transforms the study of calculus from a static exercise into a dynamic experience. This paper provides a guide for importing digital images into *GSP*, utilizing *GSP* and *TI-InterActive*! to determine equations that model the object's characteristics, determining the curve of best fit for a given object, and solving problems based on the images. Such an activity-based, technology-rich learning environment is essential if we are to capture and sustain the interest of today's students as a conceptual understanding of calculus is developed.

How Much Ice Cream Does a Sugar Cone Hold?

Determining the volume of a solid of revolution generated by revolving a right triangle around an axis is a classic calculus problem. Motivating student interest in solving such a problem is much easier when couched in terms of an actual cone and ice cream to fill it and to enjoy upon successful completion of the task. Students are given a sugar cone and a ruler and asked to devise a plan for determining exactly how much ice cream their cone holds. The plan must include information needed as well as how this information is to be incorporated into a solution. A digital camera image of the cone is taken and made available to students.

Students copy the image and paste it into *GSP*. The grid is turned on and the scale is adjusted to match the students' measurements. This is followed by plotting points on the slant height of the cone and having *GSP* measure the coordinates.

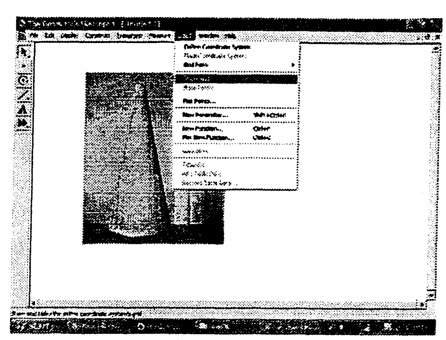

Figure 1

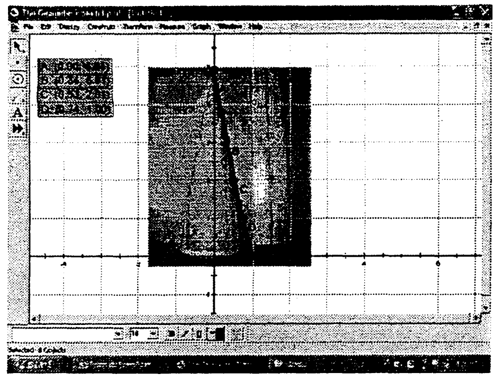

Figure 2

These coordinates are then transferred to *TI-InterActive*!'s Data Editor and a linear regression is performed using *TI-InterActive*!'s Statistics Calculation tool.

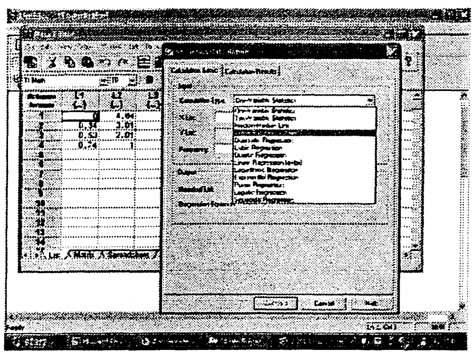

Figure 3

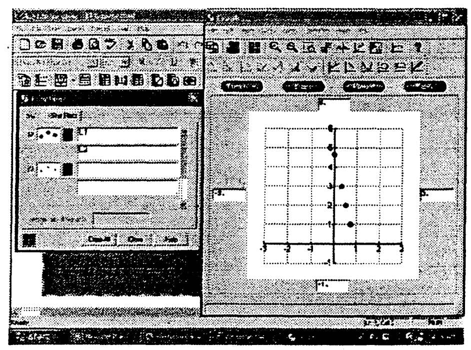

Figure 4

Once the equation of the slant height is known, students utilize *GSP*'s Plot New Function under the Graph command to superimpose the regression line on the digital image. The task now requires students to revolve the line about the y-axis. The orientation of the cone necessitates solving the linear equation for x and using y-limits for the volume integral. Students utilize *TI-InterActive*!'s Math Palette to determine the volume of the cone, noting that the cone curves upward from the x-axis where the linear equation intersects with the base of the cone. This discrepancy results in rich discussions among students.

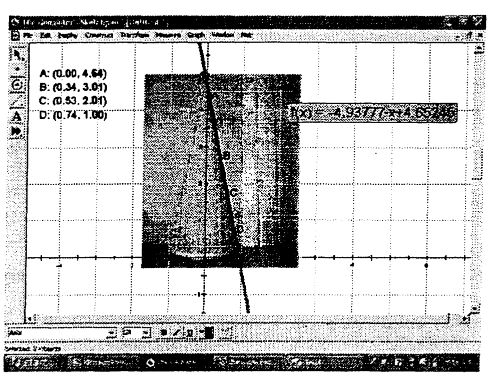

Figure 5

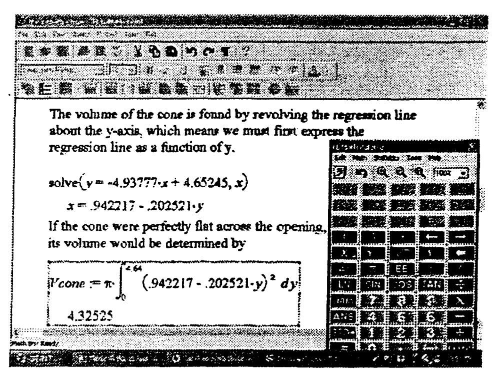

Figure 6

The curved opening is computed by plotting points in *GSP*, transferring them to *TI-InterActive!*, and determining a quadratic regression equation. After expressing the quadratic equation in terms of y, the volume of the area bounded by the curved portion of the cone and the x-axis as it is revolved about the y-axis is found using the washer method. The actual volume of the sugar cone is then calculated by taking the difference between the two computed volumes.

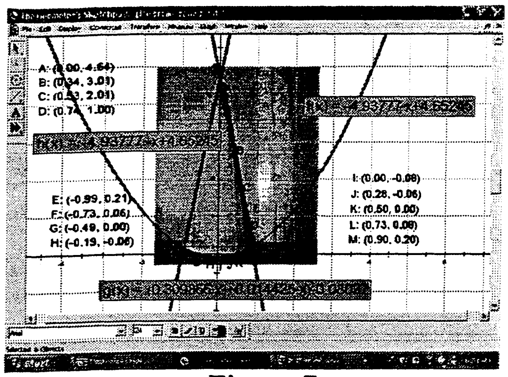

Figure 7

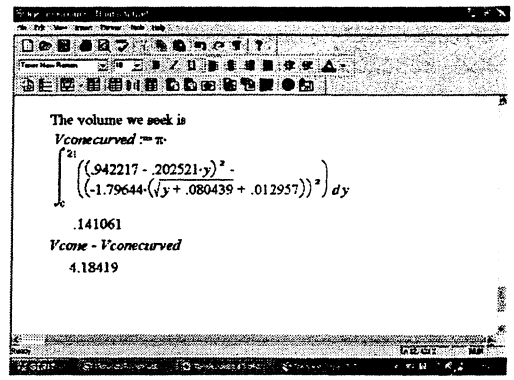

Figure 8

How Large is a Scoop of Ice Cream Atop the Cone?

Viewed from a purely theoretical perspective, an ice cream hemisphere sits atop the cone. Students are quick to respond that "real" ice cream cones contain more sphere-like scoops with double and triple scoops being the norm. However, for purposes of this problem, students compute the minimum amount that covers the opening of the cone, extending to one or more hemispherical scoops.

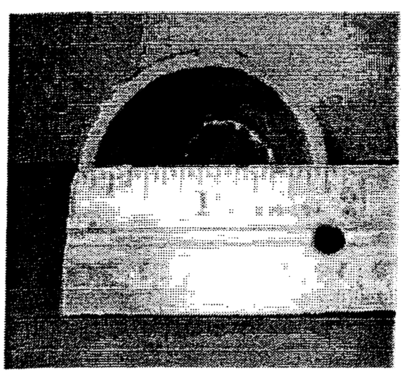

Figure 9

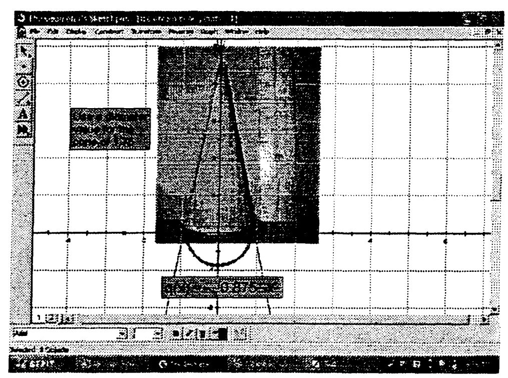

Figure 10

Students measure the diameter of the cone, determine the equation of the semicircle that fits the cone's opening, and graph the equation within the *GSP* sketch. *TI-InterActive*! is then employed to compute the volume of the hemisphere generated by revolving the semicircular region as a function of x about the x-axis. The total volume of ice cream filling and sitting atop the sugar cone is calculated.

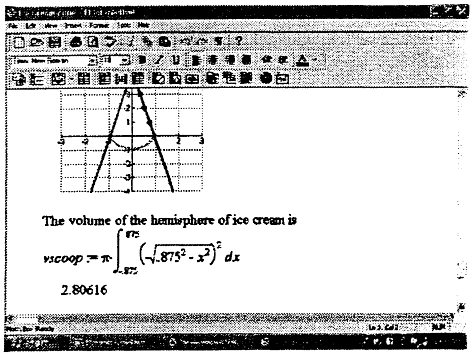

Figure 11

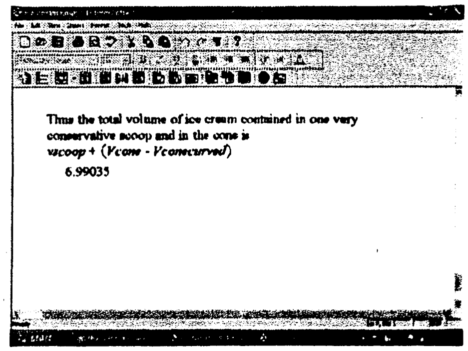

Figure 12

What is the Surface Area of a Sugar Cone?

As with the volume of the hemispherical scoop of ice cream, determining the surface area of the cone is possible using non-calculus methods. However, this activity employs classic calculus methods and *TI-InterActive!* The previously calculated linear regression equation of the cone's slant height is now used to calculate the area of the surface of revolution generated when the slant height is revolved about the *y*-axis. Content in the knowledge of the total volume of ice cream contained in the sugar cone and the area of the cone itself, calculus students begin the ice cream cone tasting session.

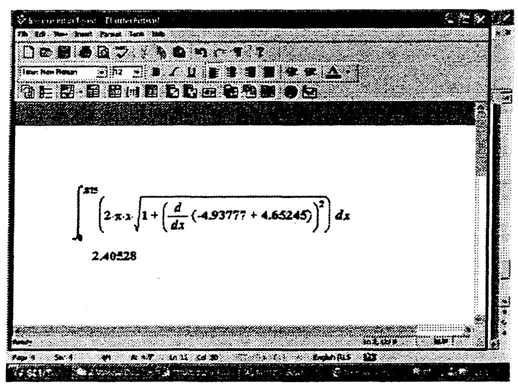

Figure 13

Conclusion

Today's calculus courses contain a broad spectrum of majors possessing a wide range of abilities. Meeting the needs of such a clientele while maintaining the integrity of the standard calculus content can easily be accomplished with tools such as a digital camera and easy-to-use software such as *The Geometer's Sketchpad (GSP)* and *TI-InterActive*! Using these tools in the teaching and learning of calculus concepts has many benefits: (a) active engagement of students, (b) greater interest in learning when concepts are tied to real-life settings, (c) flexibility in the selection of problems, and (d) promotion of camaraderie and productivity in group projects. The instructor's biggest challenge is making prudent activity selections given course meeting time restraints.