A GEOMETERS' TOOLKIT

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Whether considering traditional Euclidean topics or non-Euclidean applications in the sciences, engineering, and the arts, modern visualization and modeling technologies facilitate geometric investigation and demonstration in powerful ways. The 2002 ICTCM Computer Workshop "A Geometers' Toolkit" presented a select group of geometry technologies suitable for use at the K-12 and undergraduate levels. This paper explores pedagogical issues related to their use. The technologies themselves are available at http://www.math-ed.com, a site maintained by the author for educational purposes.

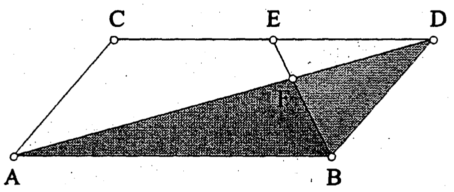
In 20th century K-12 and undergraduate level mathematics education, the discovery or invention of mathematical knowledge typically took a "back seat" to direct instruction, memorization, and recitation of concepts, facts, and skills attributed to other people. Considering the technological basis of our economy and the growing importance of graduate education, this tradition must give way to a more enlightened and productive approach in which students discover more than mathematics itself. They must discover themselves to be mathematicians. Reformers have been working for over a decade to encourage and facilitate changes of this sort in K-12 and undergraduate education. In support of this effort, powerful modeling technologies have been enlisted to motivate student interest, to facilitate observation, conjecturing, and data analysis, and to promote clear mathematical communication.

Modern Geometry

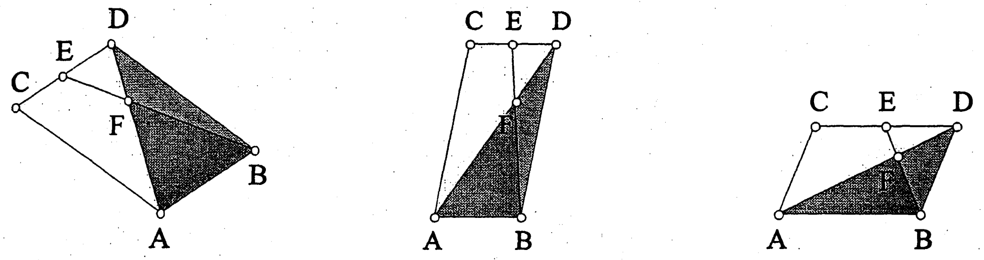
Modern Geometry (Thomas, 2002) was written with a particular audience and course in mind. Most colleges and universities offer a "Survey of Geometries" course as a routine part of their undergraduate mathematics education curriculum. Most textbooks written for this sort of course approach the study of geometry from a purely deductive frame of reference, making little or no use of modeling technologies. *Modern Geometry* provides a technology-rich, historically referenced alternative to this approach, employing both inductive and deductive mathematics throughout. With few exceptions, the technologies referenced in the *Modern Geometry* text are provided on the accompanying CD-ROM so that both teachers and students have immediate access to the technology-supported investigations in the text. As a matter of convenience, these same technology resources are available on-line at the URL http://www.math-ed.com/Resources/mgnew/.

Of all the geometry technologies mentioned in *Modern Geometry*, my favorite is *The Geometers Sketchpad, Version 4* (2002). The *Modern Geometry* CD-ROM includes over 100 GSP files and related investigations. Using this powerful, popular tool, students may quickly create 2-D geometric models, measure their features, and investigate their invariant properties. With this type of support at their finger tips, and using carefully crafted exploratory activities, all students may be led to the brink of mathematical discovery. The astonishment and satisfaction that they experience as they make their first authentic mathematical discoveries (albeit informal conjectures) is wonderful to behold. And, like true mathematicians, the experience of discovery only whets their appetite for more and for satisfying explanations. Here, indeed, is the proper motivation for proof. Table 1 shows how a series of observations made using the *Sketchpad* supports the development of well-informed conjectures.

In parallelogram ABDC, point E is the midpoint of side CD. Segment BE and diagonal AD intersect at point F. Search for relationships involving the figures polygonal areas.



In the *Sketchpad* model of this figure, points A, B, and C may be moved independently of one another to reshape the parallelogram. Each modification produces a new version of the same parallelogram. Students learn to identify invariant features of this figure by noting invariant ratios of measurements.



Version 1	**Version 2**	**Version 3**
Area DFE = 0.3 cm^2	Area DFE = 0.3 cm^2	Area DFE = 0.2 cm^2
Area DFB = 0.7 cm^2	Area DFB = 0.6 cm^2	Area DFB = 0.5 cm^2
Area FBA = 1.4 cm^2	Area FBA = 1.2 cm^2	Area FBA = 0.9 cm^2
Area CEFA = 1.7 cm^2	Area CEFA = 1.6 cm^2	Area CEFA = 1.2 cm^2
$\frac{(\text{Area DFE})}{(\text{Area DFB})} = 0.500$	$\frac{(\text{Area DFE})}{(\text{Area DFB})} = 0.500$	$\frac{(\text{Area DFE})}{(\text{Area DFB})} = 0.500$
$\frac{(\text{Area DFB})}{(\text{Area FBA})} = 0.500$	$\frac{(\text{Area DFB})}{(\text{Area FBA})} = 0.500$	$\frac{(\text{Area DFB})}{(\text{Area FBA})} = 0.500$
$\frac{(\text{Area DFE})}{(\text{Area CEFA})} = 0.200$	$\frac{(\text{Area DFE})}{(\text{Area CEFA})} = 0.200$	$\frac{(\text{Area DFE})}{(\text{Area CEFA})} = 0.200$

Table 1
Investigating Invariant Properties using the *Geometers Sketchpad*

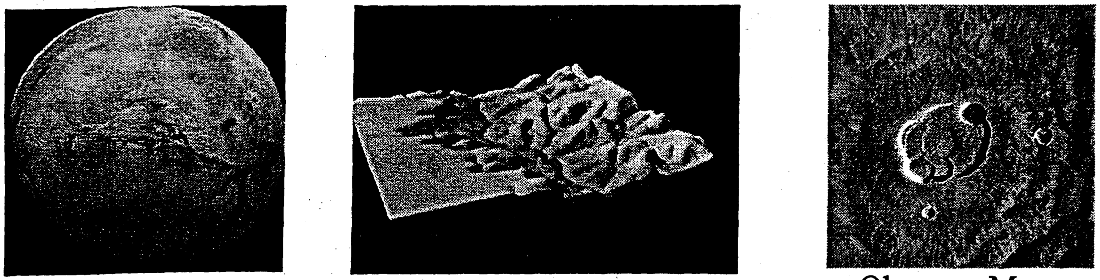
The introduction of non-Euclidean geometries comes as a shock to most students. Over the years, I have found exploratory activities into the nature and features of these spaces to be every bit as powerful and productive as *Sketchpad*-based investigations are to the development of a genuine understanding of Euclidean space. Indeed, in the absence of such experiences, formal arguments about the details of alternative geometries make little or no sense to most students. For instance, prior to the development of modeling tools for exploring hyperbolic geometry, students viewed each new fact or theorem that I presented with suspicion verging on dread. Since introducing the use of *NonEuclid* (Castellanos, 2001), a hyperbolic geometry modeling tool, students eagerly adopt the role of explorer in a strange universe, logging and comparing their observations, formulating and testing conjectures, and searching for proof.

In practice, I have found that advanced undergraduates and in-service mathematics teachers respond best to mathematical challenges posed as small-group activities. Consequently, I often divide entire classes of students into working groups of four. At the same time that I pool student talents in this manner, I "raise the bar" in terms of my expectations. Groups must submit weekly reports documenting their approach and supporting their findings. This expectation necessitates high-level mathematical communication throughout the week, as well as an equitable sharing of responsibilities. The introductory page of the *Modern Geometry Technology Resources* CD-ROM features several examples of student work. While these reports were selected for their breadth of coverage, they are not different in quality from what students give me on a regular basis when suitably challenged and technologically supported in the manner discussed in this paper. Perhaps most important, students engaged in this type of mathematical investigation gain a glimpse of the collaborative nature of mathematical research and development.

Geometry in Space

The *Geometry in Space Project* (www.cs.bsu.edu/homepages/dathomas/SpaceGrant/), supported by grants from the Indiana Space Grant Consortium and Ball State University, sought to awaken an interest in the mathematics of modeling and remote sensing among Indiana middle school and high school students. In so doing, we hoped to promote a heightened sense of the value and timeliness of geometry in today's scientific world.

The *Geometry in Space* on-line resources consist of a set of four computer-based activities and a related slide presentation by NASA/JPL Mars Odyssey Mission chief scientist Dr. Steve Saunders. The titles of the investigations are: *Orbital Mechanics: From Earth to Mars; Finding a place to land; Evaluating your landing sites;* and *Mars in Perspective.* In these activities, students learn important mathematical and scientific concepts related to the exploration of space and develop skill in the use of mathematical models and on-line scientific databases. Technologies used include the *Geometers Sketchpad, Orbital Xplorer* (Ottinsen, 2001), *NASA's Mars Explorer for the Armchair Astronaut,* and *3dem* (Horne, 2001), a terrain rendering tool that produces dramatic, realistic still and moving images of the Martian landscape. Figure 1 shows three images from these activities.



Mars Section of a Martian Canyon Olympus Mons

Figure 1

Images of Mars [Data & Images courtesy of NASA]

To evaluate the instructional value of the *Geometry in Space* materials, project leaders conducted a number of on-campus and in-school workshops for teachers and students at the middle school and high school levels. On each occasion, participants were asked to complete a brief exit survey. Among other things, this survey asked the students to rate the materials as being Interesting, OK, or Boring. In a sample of 209 responses, 74% of the students indicated that the materials were Interesting; 22% rated the materials as OK; and 4% thought they were Boring. The same students were also asked whether they would like to Keep Using the Materials (61%), Share the Materials with a Friend (35%), or Do Nothing (4%) with them.

Overall, the reaction of the students to the *Geometry in Space* materials is unambiguous: They enjoyed using the materials and would like more experiences of the same kind. The following comments were made by students on their survey response sheets:

- I enjoyed trying to orbit the earth with the rocket. Because it was fun to try different numbers to see what worked.
- I enjoy launching the rocket, because it was a challenge to see if you could orbit.
- Everything was interesting because I want to become a mathematical scientist when I am older.
- I liked flying over Mars because it looked like I was really at Mars.
- I liked everything about this program! It's awesome. You should make more of these!

Conclusion

Modeling technologies enable both novice and experienced geometers to visualize and manipulate objects in a variety of geometric spaces. I believe that this experience is enormously valuable in the development of well-founded concepts and in the evolution of grounded mathematical dialogues. Whether approached as purely mathematical pursuits or as interdisciplinary activities, students of all ages respond positively to the opportunity to experience first-hand what it means to explore and make mathematical discoveries of their own. Prior to the development of these technologies, few students of mathematics had the opportunity to experience firsthand the sort of mathematical discovery discussed in this paper. Creating and manipulating models was just too difficult and time consuming. With these technologies, every student of mathematics may experience the excitement and satisfaction of authentic mathematical discovery. This development could and should herald the dawn of a new, mathematical, age of discovery.

<u>References</u>

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