# Numerical Animation using *Mathematica*

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## 1    Introduction

When students enter an upper level mathematics class, such as Numerical Analysis, it is often times assumed that they do so with similar mathematical and computing backgrounds. However, many times the students' backgrounds, especially in computing, can be very diverse. Some students may enter this class with little or no programming experience, while others may enter the course with a strong background in Java, C++, or some other programming languages. By using *Mathematica* as the standard language in such a course, most students enter the course on a common ground. The labs and projects introduced in the following section are meant to introduce students to programming in *Mathematica* while emphasizing linear algebra concepts, such as rotations and translations, cubic splines, and animation.

## 2    Labs / Projects

The projects introduced in this section focus on creating a grid of points, fitting a cubic spline through these points, and animating this curve or set of curves. The built in spline fitting command is used:

$$<< NumericalMath`SplineFit`$$
$$SplineFit[ListofPoints, Cubic].$$

Although there are many ways to create an animation in *Mathematica*, one way is to create, or show, a list of pictures, group them together, close the subgroup of pictures leaving only the initial picture showing, and then double click on this initial picture.

### 2D Animation

The goal of these labs is to introduce through *Mathematica* the concepts of creating a list of points and using cubic splines and rotations and translations to animate. The labs include :

- creating a ball that bounces along a chosen path,

- creating a spiral, similar to the one that follows, and rotating it.
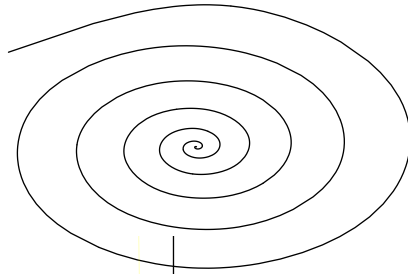
These labs as well as the full version of the 3D Animation Project explained below can be found in
[2].

## 3D Animation

The goal of this project is to combine and apply knowledge of animation and cubic splines to a 3D
animation. The following 3D grid is to be created and rotated.
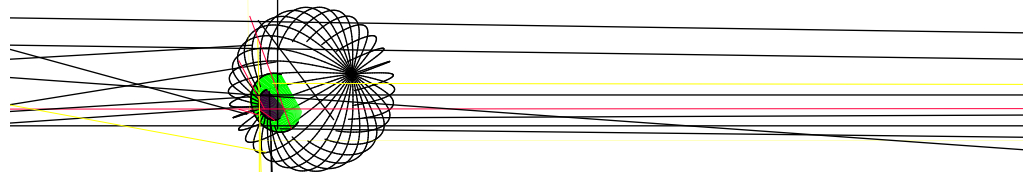
**Creating the Wire Frame Unit Sphere Using Cubic Splines**

Using spherical coordinates to describe a unit sphere, $x = \sin\phi\cos\theta, y = \sin\phi\sin\theta, z = \cos\phi$. The
following is an example of how to create one 3D circle with $\theta = 0$ as well as the rotation of this circle
by $\pi/2$.

$$pictures = \{\};$$
$$For[\theta = 0, \theta \leq \pi/2, \theta = \theta + \pi/2,$$
$$data = Table[N[\{Sin[\phi]Cos[\theta], Sin[\phi]Sin[\theta], Cos[\phi]\}], \{\theta, 0, 2\pi, \pi/16\}];$$
$$curve = SplineFit[data, Cubic];$$
$$plot1 = ParametricPlot3D[curve[u], \{u, 0, 32\}, Compiled \rightarrow False,$$
$$Axes \rightarrow False, Boxed \rightarrow False, DisplayFunction \rightarrow Identity];$$
$$pictures = Prepend[pictures, plot1]];$$
$$Show[pictures, DisplayFunction \rightarrow \$DisplayFunction];$$
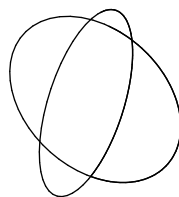
*Exercises*

1. *Create a wire frame unit sphere.*

2. *Create a 3D circle for the outer circle of the pupil and add it to your list of pictures.*

3. *Create a 3D circle for the inner circle of the pupil and add it to your list of pictures.*

4. *Load the Graphics3D package and use the Polygon command to create a separate colored polygon for the outer and inner circle of the pupil (add these to your list of pictures).*

   **Animating the Eyeball**

   6. *Add in a command that will allow the eyeball to roll along a curve. The eyeball does not need to roll in the same direction that it is rotating.*

# 3    Beyond the Basics

*Animation using cubic splines is an interesting and accessible application for Numerical Analysis students at the undergraduate level. Edwin Catmull, from Pixar Animation Studios [1], gave a talk at Mathfest 2000 about numerical methods used in animation, which first inspired me to think about animation projects in the undergraduate numerical classroom. To take the 2D animation projects to a level beyond the ideas presented here, one might consider having students animate the popular half full flour sack as shown in [3]. The next stage in 3D animation projects might involve having students create an intricate grid of a character's face in order to animate facial expression.*

# References

[1] *Edwin Catmull, 'The Mathematics of "Toy Story 2"', Mathfest 2000, invited speaker.*

[2] *Crista Coles, Numerical Animation with Mathematica, website, Elon University, URL http://math.elon.edu/~coles/ICTCM2002/ICTCM2002.ppt*

[3] *Frederic Parke, Disney Principles of Animation, website, Texas A&M University, Visualization Laboratory, URL http://www-viz.tamu.edu/faculty/parke/ends489f01/section6/princ01.html*