# THE DEVELOPMENT OF PROBLEM SOLVING ABILITY IN POST-APARTHEID SOUTH AFRICA

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## Introduction

South African society has inherited a legacy of mediocre education standards arising out of the calculated policy of the former apartheid regime to prevent access to education to non-white South Africans. Most significantly, the levels of achievement in general in mathematics, leaves much to be desired. International ranking instruments such as the TIMSS placed South Africa last among 42 countries. The most important contributor to this trend is inadequate teacher preparedness which feeds the cycle of mediocrity.

Our project attempts to address teacher deficiencies in mathematics by a programme of intensive training in the methods of solving non-routine problems. The programme comprises a short course on problem solving strategies (particularly those of Polya (2)) and their applications. While preliminary results indicate a very positive response to the course, it is quite clear that a very small number of participants is being reached. Herein lies the power of technology in offering a possible solution to the problem. In particular we examine the prospect of a web-based version of the programme.

## The Harmony Gold South African Mathematics Olympiad Teacher Development Project and History

The course consists of hands-on problem solving activities. The problems are drawn largely from the South African Mathematics Olympiad (1) which is a three round contest for two sections: Junior (grades 8 and 9) and Senior (grades 10, 11 and 12). The first two rounds are multiple choice type questions with the second round being considerably harder than the first. The third and final round consists of an essay-type paper which is below the level of the International Mathematics Olympiad (IMO). Candidates that perform well on the third round receive a call-up for the International Mathematics Olympiad Talent Search conducted from the University of Cape Town.

It is noteworthy, that while South Africa performed poorly on the Third International Mathematics and Science Survey (TIMSS) , the results of the South African team to the IMO are commendable given only 10 years of participation history. This year South Africa finished 32nd with 90 points behind Argentina with 96 points and Singapore with 112 points, Australia (26th this year, with 117 points). The South African team of six high school students won four medals (one silver and two bronze) at the 2002 International Mathematical Olympiad (IMO), held in Glasgow, Scotland, from 19 to 30 July. It is interesting to observe that South Africa outranked all Western European countries except Germany (10th), France (19th) and Britain ($27^{th}$). Of course these high achieving learners come through a rigorous programme of training. There are at least 3 week long camps prior to selecting the team.

Given the success of the IMO training project, we considered the possibility of performing something similar with teachers but starting at a lower level. It was felt that a course pitched at grade 8 and 9 level would be suitable.

## The Course Structure and Content

There are four units to the course.

*Unit 1: Problem Solving strategies*

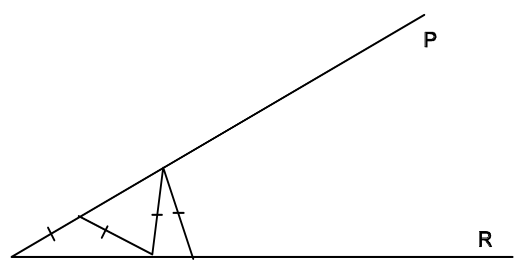
Teachers are taught well known strategies of George Polya with plenty of examples. We illustrate each strategy with a single example: (the examples should be familiar to most researchers in problem solving)

$ <u>Read and Understand the question</u>:

*I met my friend the test pilot, who had just landed at the local airport. With the pilot was a little girl, the pilot=s daughter, aged about two years. AWhat is her name?@ I asked my friend, whom I had not seen in 5 years and who had married in that time. ASame as her mother,@ the pilot said. AHello Susan@, I said to the little girl. How did I know the little girl=s name if I never saw the wedding announcement?*

$ <u>Draw a figure</u>:

*In the diagram the angle πPQR is 12 degrees, and a sequence of isosceles triangles is drawn as shown. What is the largest number of such triangles that can be drawn?*




$ <u>Look for a pattern</u>:

*What would be the third number from the left in the $89^{th}$ row of the accompanying triangular number pattern?*

1
2 3 4
5 6 7 8 9
10 11 12 13 14 15 16

$ <u>Introduce appropriate notation:</u>

*Calculate the value of (123456785) H (123456792) - (123456783) H (123456794).*

$ <u>Use logical argument</u>:

*An elimination tennis tournament was organised. There were 114 players and so there*

*were 57 matches in the first round of the tournament. In the second round, the 57 remaining players were paired, resulting in 28 matches; one player received a bye (that is, did not have to play in that round and automatically went into the next round). The remaining 29 players were then paired, and so on.*
*(a) How many matches in all were required to determine the winner of the tournament?*
*(b) Can you give the number of matches required if we were to start with another number of players?*

$ Games and Puzzles:

*In this game there are also two players, A and B, who take turns. The game uses the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. The first player must choose one of these numbers. The second player does the same and adds it to the number chosen by the first player. Then the first player chooses a number and adds it to the total, and so on. The numbers chosen need not be different. The first player to reach 100 wins.*
*What is the strategy?*

*Unit 2*: Problems from three Junior South African Mathematics Olympiad papers. Teachers are expected to work through these problems. The solutions are provided in

*Unit 3:* Additional teaching comments are contained in an **INSTRUCTOR=S MANUAL**. For example: *The solution to the following problem requires identifying the Fibonacci sequence: Each male honey-bee has a single female parent whilst each female honeybee has both a male and a female parent. In the 10$^{th}$ generation back, only, how many ancestors does a male honey-bee have?*

*(A) 89 (B) 144 (C) 10 (D) 512 (E) 233*

Further extrapolation on the Fibonacci sequence is provided including remarks on the limiting value of consecutive terms at infinity being the golden ratio.

Another example: *An ant walks around a triangle with sides 5 cm, 6 cm, and 7 cm so that it always stays 1 cm from the outside of the triangle. When it returns to its starting point for the first time, in cm, it has walked*

*(A) 19 (B) 21 (C) 24 (D) 18+π (E) 18+2π*

This example is generalised to show how the same result is valid for any convex polygon.

Unit 4: This final section consists of an assignment which is an olympiad examination paper. teachers are expected to provide full solutions and explanations.

The course structure is as follows:

This course entails a 16 hour programme of instruction in the form of
$ 8 hours of contact teaching and discussions
$ 8 hours of participants efforts

There are at least 3 contact sessions. In order to complete the course the student has to

$	complete one assignment (30% weighting)

$	discuss two problems chosen randomly by the instructor (20% weighting). One in session 2 and one in session 3.

$	write one test. (50% weighting)

To obtain the Course Certificate, a participant must achieve a minimum total score of 50%. The certificate will be awarded as a MERIT CERTIFICATE is a candidate obtains at least 75%.

## Preliminary Findings

Approximately 100 teachers were able to take the course in the first 6 months. Feedback on the programme has proved heartening in view of the apparent pessimism teachers show towards problem solving. The following preliminary results of a questionnaire were based on 55 reports already received. The questionnaire contained 12 questions probing the strength of the course and enquiring about areas for refinement. A sample of questions and responses is provided:

1. <u>Should there be a test at the end of this course?</u>

| YES | NO | UNSURE |
|---|---|---|
| 85,45% | 9,1% | 5,45% |

Comment: This question was deemed important given the negative feelings educators have developed for written tests in recent times. It was significant that the overwhelming majority of participants favoured a written assessment tool to gauge levels of understanding and competence in the course.

2. <u>Were you happy with the other assessment methods?</u>

| YES | NO | UNSURE |
|---|---|---|
| 94,54 | 3,64 | 1,82 |

Comment: The other assessment methods (besides the written test) consisted of two demonstrations, where teachers had to learn how to solve certain problems and explain their solutions to their colleagues. The instructor completed a rubric indicating the level of performance. Additionally, each teacher had to complete an assignment where detailed solutions of 20 problems had to be submitted. Again the vast majority of teachers were pleased with the assessment strategy. Respondents who did not agree felt that it was intimidating to do presentations in front of their peers. Others were pleased that the presentation forced them to understand the solutions.

3. <u>Did you find any ideas you could incorporate into your normal teaching programme</u>?

A number of teachers referred to the perceptions they had developed in reading number patterns. Some felt that ideas on the lowest common factor allowed for a more interesting introduction to the topic. A fair number appreciated some of the more practical type problems such as those involving rotating tyres on a car to obtain even wear. Teachers with a good background found the generalisations of certain geometric problems very appealing.

4. What were the shortcomings of the course?

It was felt by most teachers that the time allowed was too little to foster effective discussion of the problems. More time should be allowed for teachers to master concepts before the presentations. Many felt that the use of group-work would be advantageous.

## POTENTIAL USE OF TECHNOLOGY

Given the extent of the geographical distribution of schools in South Africa, it is vitally important to explore new avenues of bringing this type of training to teachers all over the country. Our belief is that a web-based version of the programme might be able to solve the problem. Most parts of the country now have reasonable access to the world-wide web.

From a survey on the internet, it would appear that there are a large number of problem solving websites. Nevertheless, we were not able to find any that taught skills of problem solving to adult learners such as teachers. The closest resemblance we obtained to such a scheme is http://jwilson.coe.uga.edu/emt725/EMT725.html. (3)

The challenge is to design the course in such a way that it achieves the objective of the project. In particular the method of obtaining instantaneous feedback would be very practical and motivating. However, it will be necessary to include a segment where it can be assured that the work represents the teachers own unaided effort. Additionally, the teacher demonstration aspect may not be achievable in the internet type course. This was a strong point of the traditionally managed taught course. Despite these drawbacks, efforts are being made to design the course structure in such a way as to achieve the objectives of the project: that of empowering teachers who were excluded from such exposure in the past, with the skills they need to tackle non-regular problems in mathematics. Jensen (4) utilised children playing the role of the teacher as a means of developing problem solving ability. We extrapolate this same technique to teachers among their peers. Of course, the more serious spin-off is an improvement in the problem solving skills of learners. It is desirable to study the impact of the development course on the achievement of candidates from schools where teachers have indeed taken the programme and to contrast this with schools where such exposure has been missing. This will be the subject of future research. At present the Mathematics Olympiad and the professional association of teachers have websites with contest questions and solutions (5) and (6).

## Conclusion

We have traced briefly the poor achievement levels of South African students in international testing back to the policies of the past apartheid style government. Such inadequacies are surfacing now when the international competitiveness of South Africa is coming to the fore. Our project, to develop the problem solving skills of teachers in a systematic way, was discussed. Preliminary research has shown a positive effect of the training, in particular an improvement in the attitude, knowledge and skill of teachers. We have also indicated the potential of a web-based version of the course, principally to achieve a wider geographical coverage.

## References


(1) Heideman, NJ et al (Editors) (1995), *Invitation to the South African Mathematics Olympiad*, South African Academy of Arts and Science, Pretoria, ISBN 0949976334

(2) Polya, G. (1973). *How to solve it.* Princeton, NJ: Princeton University Press. (Originally copyrighted in 1945).

(3) Wilson, P. S. (Ed.)(1993). *Research Ideas for the Classroom: High School Mathematics.* New York: MacMillan.

(4) Jensen, R. (1984). A multifaceted instructional approach for developing subgoal generation skills. Unpublished doctoral dissertation, The University of Georgia.

(5) www.amesakzn.org (website of the Association for Mathematics Education of South Africa - Province of KwaZulu Natal)

(6) http://science.up.ac.za/samo/ (website of the South African Mathematics Olympiad)