Demos with Positive Impact: A Project to Connect Mathematics Instructors with Effective Teaching Tools

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Introduction

Demos with Positive Impact is an NSF project to develop a web-based collection of effective instructional demonstrations and to connect this resource to mathematics instructors. In this article we describe the project and showcase some of the demos that are in the Demos with Positive Impact collection. In addition, we provide an invitation for participation in the project.

Demos: Helping the Instructor to Facilitate Learning

As instructors we use various tools to engage students on a level beyond the instructor's dialogue. In any instructional setting, the role of the instructor is that of facilitator of learning. Demos provide a description or explanation of an idea or concept. Often utilizing some type of instructional technology ranging from computer software to physical objects or props, demos are vignettes that are incorporated into a classroom presentation. These vignettes are important tools that are appropriate (and necessary) for any instructional setting.

Demos with Positive Impact, funded by an NSF proof-of-concept grant, focuses on collecting, developing, and disseminating demos that our colleagues have found to be effective classroom tools. This project is different from other educational materials development projects in that it focuses on what the instructor does to facilitate learning, rather than activities done by students. The project has a broad scope, encompassing topics from college preparatory mathematics to post-calculus undergraduate mathematics courses. The Demos with Positive Impact collection is a valuable resource for pre-service and in-service secondary teachers as well as undergraduate mathematics instructors (both inexperienced and experienced). These demos are adaptable to a variety of learning styles and teaching environments. In addition, efforts are being made to provide versions for various technology platforms, including platform-independent variations that run in a web browser.

The success of this project depends on tapping a largely unharvested source: the collective experience of our colleagues across the country and beyond. In this article, we showcase some of the ideas that several of our colleagues have shared with us.

Taylor Polynomials—A Visual Approach to Approximations

An important area in mathematics is the computation of approximate values for functions at particular points. One of the first encounters students have with such approximations is when they use the slope of a secant line to a graph to estimate the slope of a tangent line. Then the equation of a tangent line at a point is a linear approximation to the function in a neighborhood of the point. Later on, as students study Taylor Polynomials the more general problem of approximating a function by a polynomial is encountered.

Suppose we wish to approximate a function y = f(x) in the neighborhood of x = a by a polynomial function of the form

$$p_n(x) = a_0 + a_1(x-a) + a_2(x-a)^2 + \cdots + a_n(x-a)^n .$$

The strategy we use to find the coefficients is to require a high degree of "match" at x = a. Specifically, if we require that the polynomial and its first n derivatives at x = a match the function and its first n derivatives at x = a, the result of these requirements is that we construct a formula for the n^{th} Taylor Polynomial for f, centered at x = a:

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n .$$

When a = 0, the polynomial is called an n^{th} Maclaurin polynomial for f.

The rather complicated formula can tend to obscure the geometric significance of the approximating function found in this manner or how good the approximation might be. This demo provides Javascript slide shows illustrating the graph of *y* = f(*x*) together with Taylor Polynomials of increasing order. The slide shows allow students to get a visual sense of the meaning of “approximating function.” It also gives a very intuitive way to discuss the “interval of convergence.” In addition to the Taylor Polynomials, there is an option by which the error is investigated from a graphical point of view. The graphs illustrate how the error in the approximation about x = a decreases as the order of the Taylor polynomial increases. Figure 1 shows a sample of the type of graphs included within the Javascript slide shows.

Although the Javascript slide show is not interactive, a gallery of illustrative examples is provided. These have the advantage of being platform independent. Additionally, Mathematica, Maple, Mathcad, and TI-89 calculator routines are given so that instructors can generate approximation visualizations for functions of their choice.

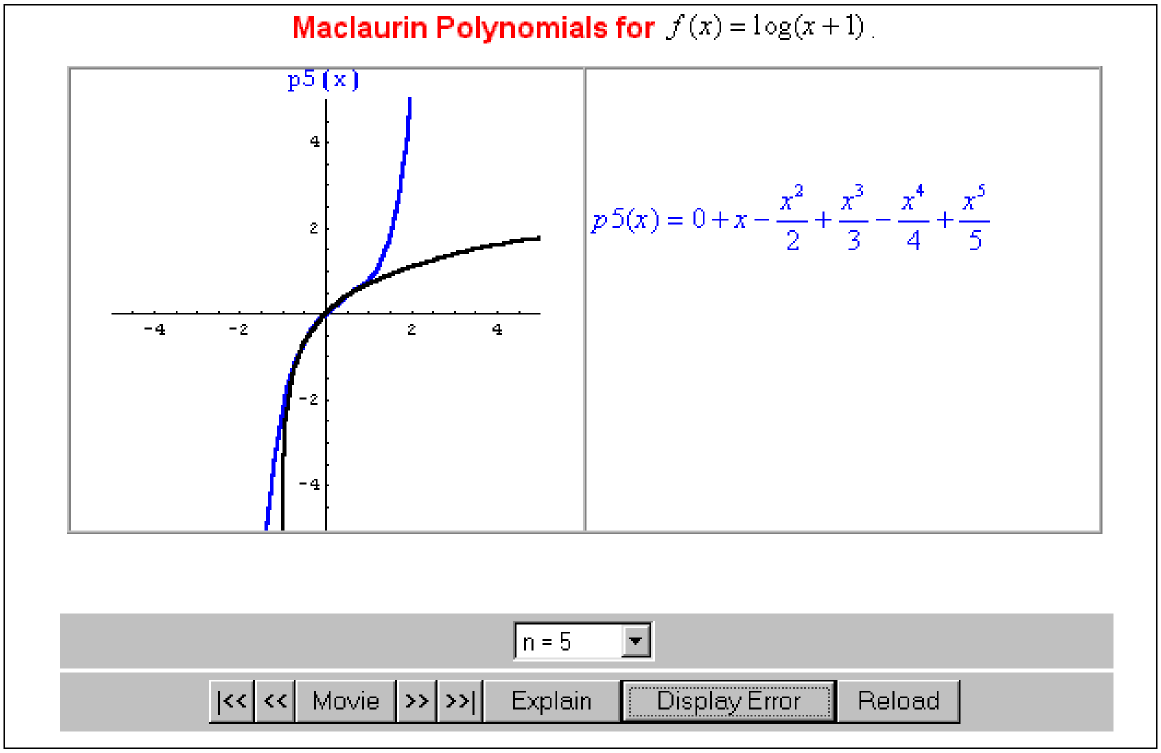
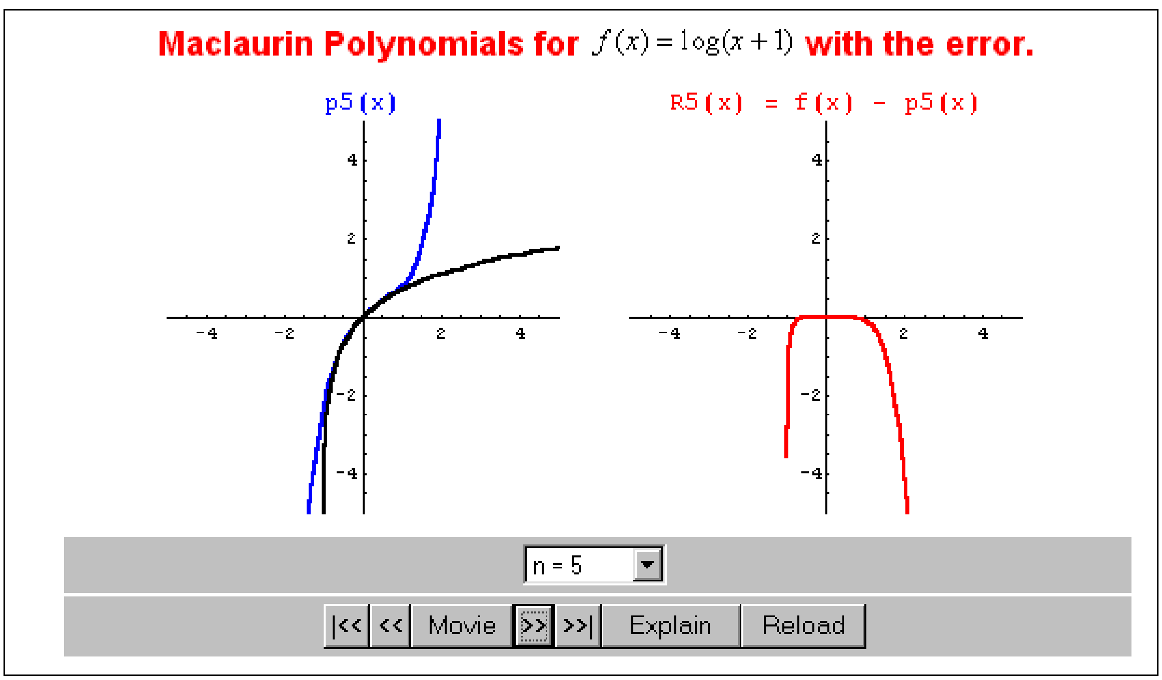


Figure 1. Maclaurin Polynomial Approximation



(c) Maclaurin Polynomial and the Error
Figure 1. Taylor Polynomial Demo

The Vigenere Cipher

A cipher is a secret method of writing, as by code. **Cryptography**, in a very broad sense, is the study of techniques related to aspects of information security. Hence cryptography is concerned with the writing (ciphering or encoding) and deciphering (decoding) of messages in secret code. Such considerations emphasize privacy and confidentiality of information. However, in today's electronic society cryptography has developed to encompass other features such as data integrity and authentication. Cryptography has evolved into a mathematical discipline that plays a major role in business and government.

Cryptography has a long and rich history dating from the Egyptians some 4000 years ago and there are many interesting encoding schemes. A **substitution cipher** is an encoding process that maintains the order of the letters in the message, but changes their identity. Another letter or symbol replaces each letter of the message. For example, Morse code is a substitution cipher in which each letter is replaced by a specific set of dots and dashes. Many substitution ciphers use only one alphabet, and are called "monoalphabetic". This means that we substitute one and only one letter for a particular letter in the message. For example, the same substitute letter or symbol replaces every T in the message. Such a cipher scheme is easy to remember, but is also vulnerable to "cracking" using frequency analysis (letter counting). Given a sufficiently large encoded message derived using a monoalphabetic substitution cipher, it can readily be "cracked" by comparing the frequency of letter occurrences in the coded message with the frequency of letter occurrences in the language used for the message.

In order to make substitution ciphers more secure, more than one alphabet can be used. Such ciphers are called **polyalphabetic**, which means that the same letter of a message can be represented by different letters when encoded. Such a one-to-many correspondence makes the use of frequency analysis much more difficult in order to crack the code. We describe one such cipher named for Blaise de Vigenere a 16th century Frenchman.

The **Vigenere cipher** is a polyalphabetic cipher based on using successively shifted alphabets, a different shifted alphabet for each of the 26 English letters. The procedure is based on the tableau shown in Figure 2 and the use of a keyword. The letters of the keyword determine the shifted alphabets used in the encoding process.

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Figure 2. A Vigenere Cipher Tableau

The demo entitled The Vigenere Cipher illustrates how the Vigenere cipher can be used to encode and then decode a message. The web site includes several links to Java applet implementations of the cipher, but also includes a Visual Basic routine as well as a MATLAB implementation that may be downloaded. In addition, some suggestions are given to illustrate how the cipher might be used within a classroom setting.

Modeling the Spread of Disease—A Logistic Model

The logistic curve is used to model a variety of physical situations in which a quantity's growth is "self-limited," that is, the growth rate of the quantity depends on the size of the quantity in such a way that if the quantity grows beyond a certain level, the growth rate decreases. The logistic model nicely describes the behavior of certain types of growth in business, economics, populations, the spread of disease, and sales forecasts. In situations of "controlled growth" the logistic curve frequently provides an easily constructed graph that is readily understood. For example:

- Currently, our economy is continuing to grow, but at a slower rate.

Often this and similar statements are misunderstood, but can be clarified with pictures and the discussion of the logistic curve.

The story goes that Sam Walton of Wal-Mart fame made his money by having inventory reported every night. As soon as products were being bought at a decreasing rate he would stop stocking that product. Thus there was never a surplus of inventory that needed to be sold at a low price. This short tale has made many business students stop asking why they take calculus.

The Logistic Curve Demo is a simple yet effective demo that illustrates how a simulation of the spread of a communicable virus leads to a logistic curve. The demo involves the entire class; the disease progresses with one person becoming ill and the virus spreads in a random manner from the infected person to others in the class. The disease continues until the entire population has contracted the virus. Figure 3 illustrates the result of a simulation involving a class of 25 students in which it took 8 days for all of the students to become ill.

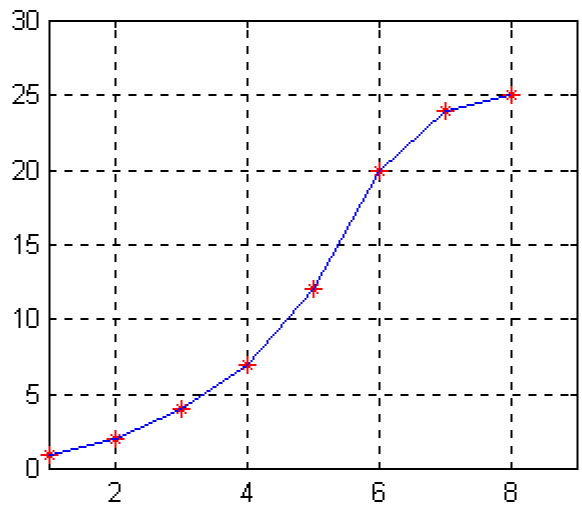


Figure 3. Results of the Simulation of the Spread of a Virus

This demo has been used with great success in a variety of classes, including pre-calculus, business calculus, and calculus classes. Students are involved in the process to generate the information used to explain a behavior that appears in many applications. They seem to personalize the experience provided by this demo and are better able to explain such behavior when they meet it in the future.

This demo shows how the TI-89 and TI-83 calculators can be used for random number generation. In addition, MATLAB and Mathematica implementations of simulations are included.

An Invitation to Participate

Every mathematics teacher has his or her own private toolbox of instructional ideas. The **Demos with Positive Impact** project recognizes that faculty members are often not encouraged nor rewarded for activities related solely to development of pedagogical tools, so while we cannot pay contributors for their ideas, we can provide a mechanism by which mathematics instructors can get some recognition for innovative teaching strategies. We invite you to visit the growing **Demos with Positive Impact** web site for an on-line opportunity to contribute to the project. In addition, we welcome feedback on the collection or on individual demos within the collection.

GOT DEMOS?

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