Math 140 with Group Projects
Spring 1999

Lab 2: Finding Absolute Extrema on a Closed Interval Domain

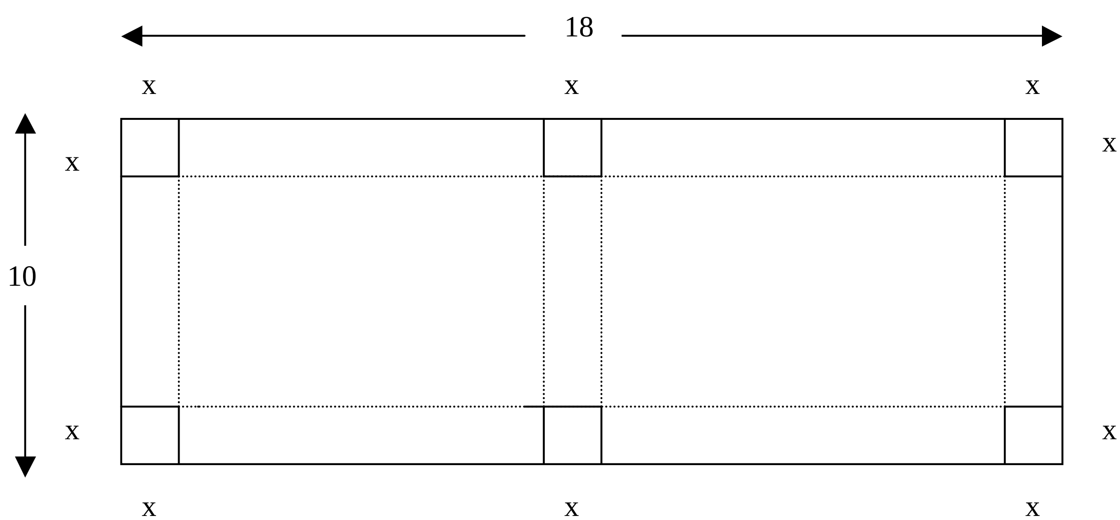
Goals

- To write a function to be optimized
- To determine a feasible domain for a function to be optimized
- To investigate a function's absolute extrema graphically
- To determine a function's absolute extrema analytically

Group Lab Problems

Many calculus problems involve finding the greatest or smallest value (or values) that a function assumes over an appropriate domain. These greatest or smallest value points are called global or absolute extrema. In this lab, for each problem you will write a function (called a primary equation) that is to be optimized. You will determine an appropriate domain for which the problem makes sense. First you will graphically investigate absolute extrema of that function on the closed interval domain. Then analytically you will solve for the absolute extrema of the function. Draw complete graphs, labeling axes and showing the window settings on your graphing calculator. Round off decimal approximations to five decimal places.

1. Pizza House just introduced a larger personal size pizza. The Pizza House Company needs a design for a box to package this new pizza. The box is to be made from a piece of cardboard of size 10 inches by 18 inches. The company will generously reward the employee team that designs the box with the largest volume. The design must be simple, as shown below. Six squares of width x are to be cut from the cardboard, which will then be folded into a box of height "x". Your team is challenged to determine the size of x that will maximize the volume of the box.



a. Find all dimensions of the box, in terms of x. Write a function that represents the volume of the box. This function is the primary equation to be maximized. Determine an appropriate closed interval domain for the problem, based on the size of the cardboard.

b. Graphically investigate the absolute extrema of the function on the closed interval domain. Approximate the location of the extrema value(s).

c. Analytically solve for the absolute extrema of the function. Use calculus to find the critical number(s) of the function. Evaluate the primary equation function at the endpoints of the domain and at the critical number(s) of the function. Find the dimensions of the box that has the maximum volume.

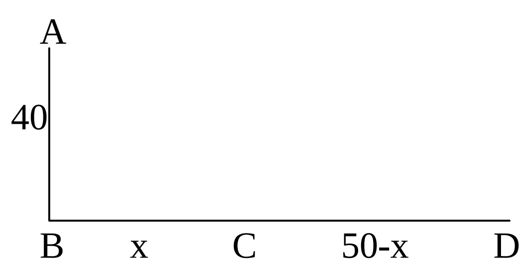
2. Cheap Pop, a new Fortune 500 company, plans to sell soda pop in 12-ounce cylindrical cans. Cheap Pop will generously reward the employee team that designs the can that can be constructed with the least amount of material. Your team is challenged to determine the dimensions that will minimize the surface area of the can. (Your team knows that 12 liquid ounces is approximately equal to 355 cubic centimeters.)

a. In order to represent the surface area of the can as a function of r, you will first need to write a secondary equation that represents the volume of the can. Solve the volume equation for h, in terms of r. Substitute the expression for h into the surface area function. The surface area function is the primary equation to be minimized. Determine an appropriate closed interval domain for the problem.

b. Graphically investigate the absolute extrema of the function on the closed interval domain. Approximate the location of the extrema value(s).

c. Analytically solve for the absolute extrema of the function. Use calculus to find the critical number(s) of the function. Evaluate the primary equation function at the endpoints of the domain and at the critical number(s) of the function. Find the dimensions of the can that has the minimum surface area.

3. Dune Buggy Annual Timed Races are held at Parched Desert. Your team driver starts in the desert at point A, located 40 miles from point B. As shown below, points B and D lie on a straight road. Your driver can travel at 45 mph on the desert and 75 mph on the road. Your team will win this prestigious race if your driver arrives at the finish line, located at point D, in 85 minutes or less. The distance between points B and D is 50 miles. Your team is challenged to determine the route that will minimize the time of travel to win the race.



a. Suppose your driver travels to point C located x miles down the road from point B, as shown above. To minimize time, write an equation that represents time as a function of distance and rate. This function is the primary equation to be minimized. Determine an appropriate closed interval domain for the problem.

b. Graphically investigate the absolute extrema of the function on the closed interval domain. Approximate the location of the extrema value(s).

c. Analytically solve for the absolute extrema of the function. Use calculus to find the critical number(s) of the function. Evaluate the time function at the endpoints of the domain and at the critical number(s) of the function. Describe the route that will minimize the total driving time.

Acknowledgments: Problems 2 and 3 were adapted from Calculus, Gerald L. Bradley and Karl J. Smith, Prentice Hall, 2nd Edition, 1999.