

Solving a nonlinear optimization problem using Excel

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Introduction

In this paper, we discuss how the Excel spreadsheet can be used to solve a nonlinear optimization problem arising in the steel industry. The production of steel uses continuous casting as an intermediate process. An optimization problem can be formulated for the production variables involved. A theoretical analysis was performed in [1]. We will examine how the problem can be implemented in a spreadsheet and how it can serve as a useful teaching tool in a mathematical modeling course. The entire problem is given in the Excel workbook `steel.xls` [2].

Problem Statement

The cast product is characterized by its dimensions, THICK (thickness), WID (width) and LEN (length). One can achieve different objectives in the production process. In this paper, we will consider maximizing the output, calculated in tons per hour (TPH).

Variables in Problem

TPH	rate at which the cast product comes off the caster (tons/hr.)
THICK	thickness of cast product (ins.)
WID	width of cast product (ins.)
LEN	length of cast product at the time it solidifies (ft.)

Objective

We wish to maximize the output (TPH)

The equation for TPH in terms of the dimensions of the cast product is given as follows. The derivation is given in [1].

$TPH = (1.44\rho k^2)LEN*WID/THICK$, where

ρ : density of steel k : solidification factor of steel

Constraints

- The aspect ratio, WID/THICK, is bounded above
- The thickness, THICK, must have a lower limit
- The cross-sectional area, LEN*WID, must be between prescribed lower and upper limits
- The length, LEN, is bounded above

Mathematically, the constraints can be written as follows, with the items in blue italics representing constants:

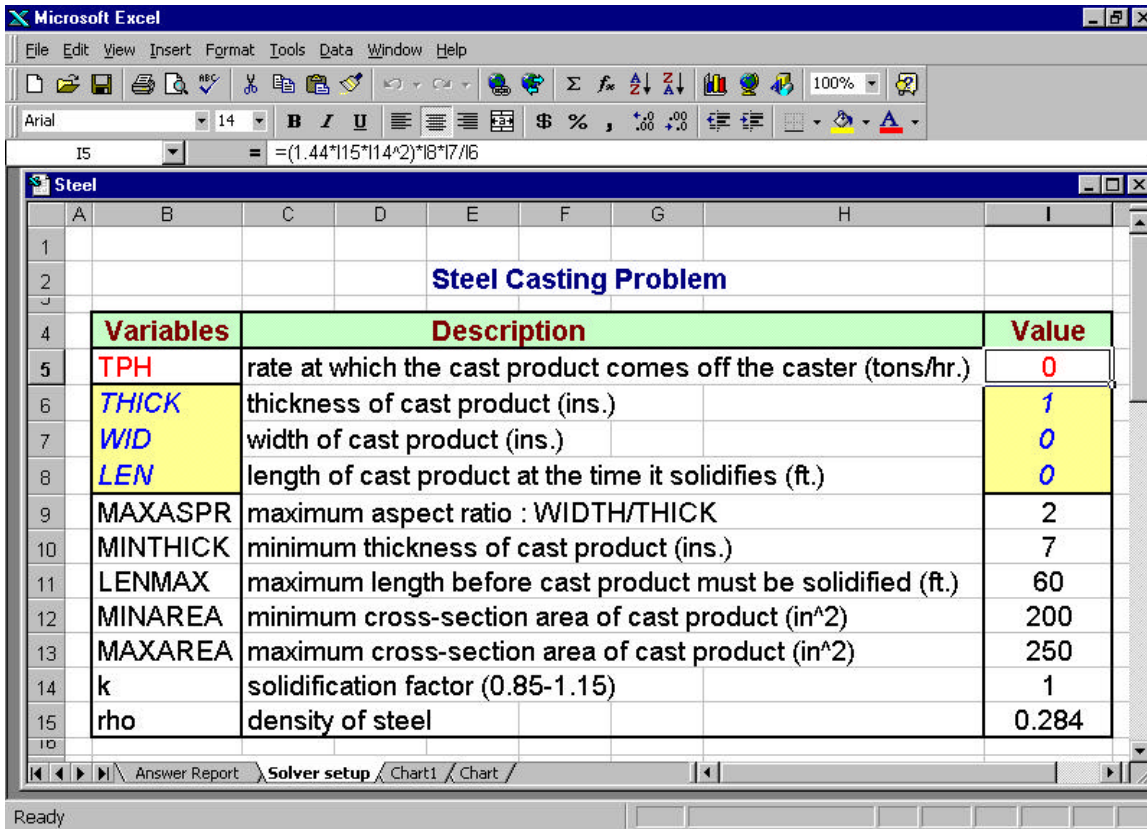
$WID/THICK \leq \textit{Max Aspect Ratio}$

$\textit{Minthick} \leq THICK$

$\textit{Minarea} \leq WID*LEN \leq \textit{Maxarea}$

$LEN \leq \textit{Lenmax}$

The problem can be set up in Excel as shown in the figure below. The complete problem setup is given in the accompanying workbook steel.xls.



The objective and constraints are then entered as follows:

Note that the formula for the following objective is highlighted in the figure above.

Maximize output in
tons/hour

$$TPH = (1.44 \cdot \rho \cdot k^2) \cdot LEN \cdot WID / THICK$$

given the following constraints

Aspect ratio \leq Max Aspect Ratio

0 2

THICK \geq MINTHICK

1 7

Area \geq MINAREA

Excel cells containing
constraints

0	200
---	-----

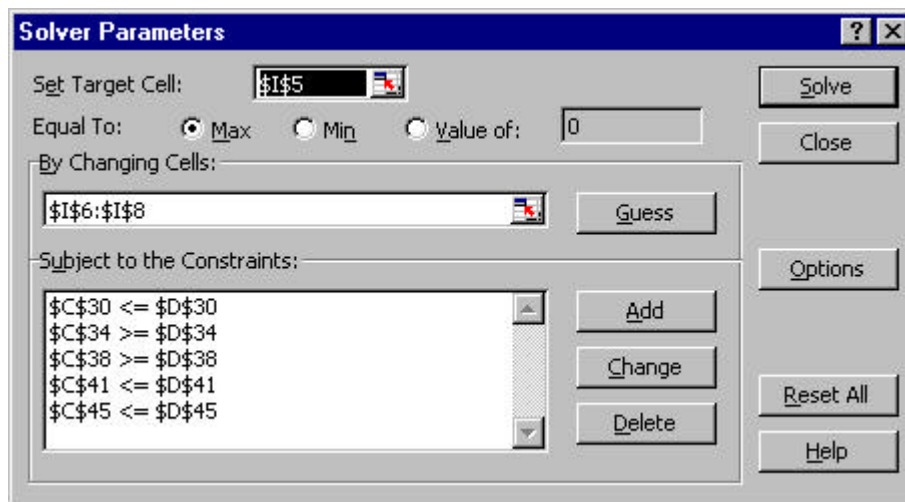
Area <= MAXAREA

0	250
---	-----

LEN <= LENMAX

0	60
---	----

The Solver dialog box is then called up from the Tools menu bar. All constraints, variables, and constants are entered through cell references. The following figure shows the dialog box corresponding to this problem.



Note that the objective function is in the **target cell** which is set to **maximize**.

The other cell references are from the constraints listed in the cells.

Clicking the Solve button will bring up another dialog box which will ask for a variety of options. One of these is an answer report, which is useful to generate and given in the following page.

Microsoft Excel 8.0e Answer Report
Worksheet: [steel.xls]Sheet1
Report Created: 6/14/99 11:33:00 AM

Target Cell (Max)

Cell	Name	Original Value	Final Value
\$I\$6	rate at which the cast product comes off the caster (tons/hr.) Value	0	49.07520012

Adjustable Cells

Cell	Name	Original Value	Final Value
\$I\$7	Thickness of cast product (ins.) Value	1	9.999999983
\$I\$8	width of cast product (ins.) Value	0	20.00000001
\$I\$9	Length of cast product at the time it solidifies (ft.) Value	0	60

Constraints

Cell	Name	Cell Value	Formula	Status	Slack
\$C\$26	Aspect ratio <= Max Aspect Ratio	2.000000005	\$C\$26<=\$D\$26	Binding	0
\$C\$30	THICK >= MINTHICK	9.999999983	\$C\$30>=\$D\$30	Not Binding	2.999999983
\$C\$34	Area >= MINAREA	199.9999998	\$C\$34>=\$D\$34	Binding	0
\$C\$41	LEN <= LENMAX	60	\$C\$41<=\$D\$41	Binding	0
\$C\$37	Area <= MAXAREA	199.9999998	\$C\$37<=\$D\$37	Not Binding	50.0000002

It is useful to ask students to analyze the answer report. In particular, they can examine which constraints are binding and which are not binding. They can vary the values in the constraints and observe what happens to the optimal solution. The reference [1] discusses various scenarios which can be easily explored using Excel.

Conclusion

We have presented an example of a nonlinear optimization problem which can be solved using Excel. The computational ease of the spreadsheet lends itself to discussion of various scenarios, thus yielding insight into the problem. Students can therefore see the value in mathematical modeling in a real world setting.

Reference

[1] D. Yan, "Optimal and Continuous Strategies for Determining Continuous Caster Product Dimensions", *SIAM Review*, Vol 39, No. 3, pp. 461-471, Sept. 1997.

[2] R. Narasimhan, Excel Workbook for the Steel Casting Problem,
http://www.spc.edu/~narasimhan_r/m382/excel_files.html