NEWTON METHOD and HP-48G

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I. Introduction

Newton method is an often-used procedure to find the approximate values of the solutions of an equation. Now, it is covered by most textbooks of Calculus as an application of derivative.

Its basic idea is to use the intersection point of the tangent line with X-axis to approximate the solution of the equation. This idea is described by following iterative formula:

Initial guess: x_0

Iterative formula: $x_{n+1} = x_n - \dfrac{f(x_n)}{\frac{d}{dx} f(x_n)}$ $\quad \frac{d}{dx} f(x_n) \neq 0 \quad$ n=0,1,2,...

When using Newton method, usually we follow 3 steps:

Step 1: Locate the solution of the equation and make a good initial guess;

Step 2: Compute the derivative of f(x) and set up the iterative formula;

Step 3: Perform the calculation to find approximate values of the solution up to the required accuracy.

It should be emphasized that the Step 1 is difficult and significant since it is hard to determine where the solutions are and to make a good guess from an equation and moreover the sequence of approximate values may not converge if the initial guess is sellect blindly. And, sometime the Step 2 is formidable if f(x) is complicated. Moreover, the Step 3 is always a time-consuming job.

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However, the graphing calculator will help us to overcome the difficulty which exits in Newton method.

II.Approach of graphic calculator(HP-48G) for Newton method

The approach of graphic calculator for Newton method is as follows:

1.Use "Plot application" of HP-48G to graph the equation.

The graph of the equation provides us with a viewable image so that it is easy to locate solutions of the equation and the "trace" function of HP-48g helps us to get a good initial guess of the desired solution .

2.Use the program "ITF" to set up the iterative formula.

This program can be produced by following keys in HP-48G.

Left Shift `<< >>, Left Shifi Stack NXT DUP, x , d/dx , / , x ,

Left Shift SWAP , - , ' N F , STO .

This program will take a function from the first stack and find its derivative and then set up the iterative formula and store it in the variable 'NF'.

3.Use the program "APV" to compute the approximate values up to desired accuracy.

This program can be produced by following keys in HP-48G.

Left Shift << >> , N F , Left Shift ->NUM , Left Shift Stack NXT DUP ,

' x , STO .

This program will take a number stored in the variable 'x' to compute the successive approximate value and put it the first stack and store in 'x" for " computing next approximate value.

Now, Let's see an example to illustrate this approach.

Find: first 6 approximate values of positive solution of $x^3+1=\frac{3}{2}\cdot\sin(x+1)$

Consider the function: $f(x) := x^3 + 1 - \frac{3}{2}\cdot\sin(x+1)$, zeros of f(x) are solutions of equation.

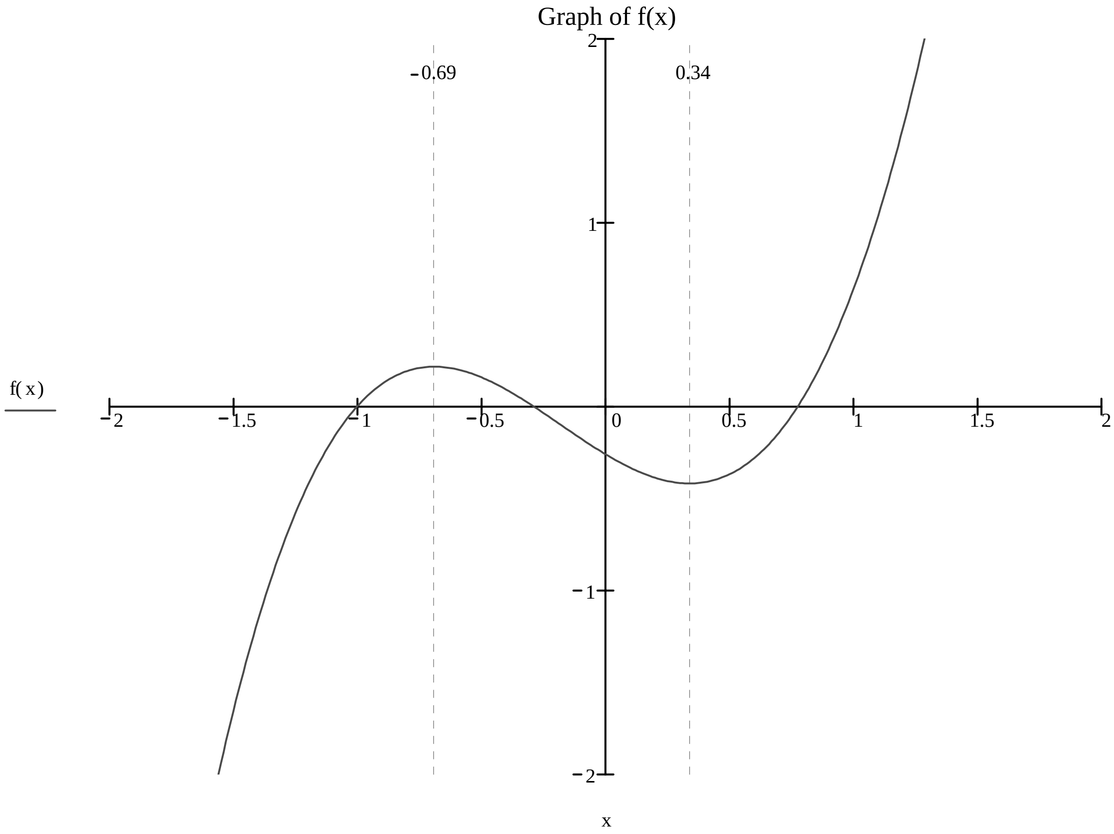
From the graph below, it is easy to see that f(x) has 3 zeros:

$x := -0.9$ $r_1 := \mathrm{root}(f(x), x)$ $r_1 = -0.999639$

$x := 0$ $r_2 := \mathrm{root}(f(x), x)$ $r_2 = -0.293265$

$x := 0.6$ $r_3 := \mathrm{root}(f(x), x)$ $r_3 = 0.776657$

$x := -2, -1.99 .. 2$



It is obvious that the approximate values of r_3 are our desired."

Then, use the program "ITF" to set up its iterative formula.

$$f1(x) = \frac{d}{dx}\left(x^3 + 1 - \frac{3}{2}\cdot\sin(x+1)\right) \qquad f1(x) := 3\cdot x^2 - \frac{3}{2}\cdot\cos(x+1)$$

$n := 0..6$ $\quad x_0 := 0.6$ $\quad x_{n+1} := x_n - \frac{f(x_n)}{f1(x_n)}$ $\quad x_{n+1} := x_n - \frac{(x_n)^3 + 1 - \frac{3}{2}\cdot\sin(x_n+1)}{3\cdot(x_n)^2 - \frac{3}{2}\cdot\cos(x_n+1)}$

Finally, take initial guess $x_0 = 0.6$ and use the program "APV" to find 6 approximate values of r_3.

To check the accuracy, define the error : $E_n := x_{n+1} - x_n$, The results are :

$n =$	$x_n =$	$E_n =$
0	0.6	0.252145
1	0.852145	-0.068504
2	0.783641	$-6.983143\cdot10^{-3}$
3	0.776658	$-7.091283\cdot10^{-5}$
4	0.776587	$-7.282478\cdot10^{-9}$
5	0.776587	0
6	0.776587	0

III. Approach of graphic calculator for grading

When grading students' paper about Newton method, the work load becomes very heavy because students take different initial guess and get different approximate values.Now, this approach can save our time and labor.Simplely store students' initial guess in variable 'x" and press 'APV' key to get approximate values. Then, we can use them to check students' calculation.

For example, if a student selects his initial guess $t_0 := 0.9$ then we get :

$t_0 := 0.9$

$$t_{n+1} := t_n - \frac{(t_n)^3 + 1 - \frac{3}{2}\cdot\sin(t_n+1)}{3\cdot(t_n)^2 - \frac{3}{2}\cdot\cos(t_n+1)}$$

$t_n =$
0.9
0.793806
0.777
0.776587
0.776587
0.776587
0.776587