Paper

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Title:

Calculator/Geometry Strategies for Understanding $x^{m/n}$.

Abstract:

Some students may benefit in the understanding and applying real numbers of the form $x^{m/n}$ from a combination of calculator and theoretical geometric techniques. This paper will demonstrate a seven-step calculator routine and a spiral sequence of similar right trigons for conceptualizing and applying real numbers of the form $x^{m/n}$, with examples. Papers will be distributed which will include specific geometric spiral examples with corresponding coordinates and calculator routines, and a rationale for such spirals will be attached.

Paper:

Quantum jumps in cognitive development/maturation with respect to understanding and applying real numbers seem to occur in transitions from integers to rational numbers and from rational numbers to irrational numbers. The focus of this paper is upon enhancing the transition from rational to irrational numbers, particularly, irrational numbers expressible as $x^{m/n}$. The specific emphasis is upon assisting those students whose learning styles compliment the combination of using a calculator and theoretical geometrical models.

The calculator technique for representing $x^{m/n}$ is a simple seven-step routine; $C\left(x^{m/n}\right)$: ON, x, y^x, m, $/$, n, $=$ ____ ; which renders a decimal approximation for $x^{m/n}$ (a seemingly rational expression). The calculator as an educational tool strategy may be satisfying for some students in isolation, but is probably not satisfying, by itself, for all students.

A theoretic geometric approach for comprehending $x^{m/n}$, as proposed in this paper, is a sequence of similar right trigons which construct a spiral (perhaps like an unfolding fan of right trigons). The sequence of similar trigons is constructed by requiring the terminal edge of a preceding trigon to be congruent and coincident with the initial edge of the subsequent trigon. The initial and terminal edges of a trigon are mutually perpendicular and are each measured by a ruler. The first trigon of the sequence has an initial edge of one unit and a terminal edge of x units (for the purposes of this paper x is greater than one). A typical example of such a sequence or spiral (or fan) is depicted in the first diagram. Similar to the observation about calculator results in isolation, although this geometric strategy in isolation may be somewhat satisfying to some students, the strategy is probably not satisfying, by itself, for all students.

The idea is to use a combination of calculator results and geometric results to enhance the understanding of irrational numbers of the form $x^{m/n}$.

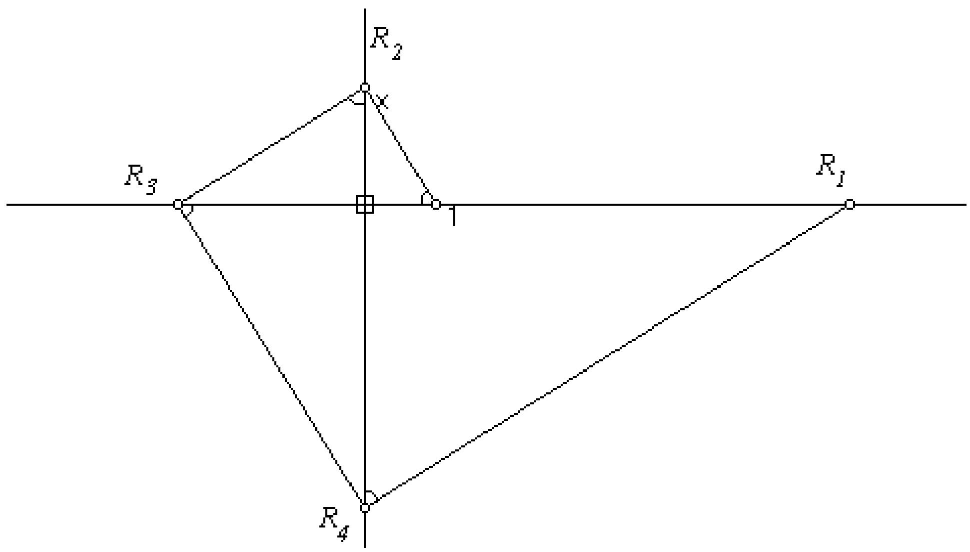


Diagram 1. A typical spiral of similar right trigons.

The spiral of similar right trigons, depicted in Diagram 1, yields the sequence:

$$\frac{1}{x^1} = \frac{x^1}{x^2} = \frac{x^2}{x^3} = \ldots = \frac{x^{n-1}}{x^n},$$

or the sequence of terminal edges:

$$x^0, x^1, x^2, x^3, \ldots, x^n, \ldots,$$

which is depicted in diagram 2.

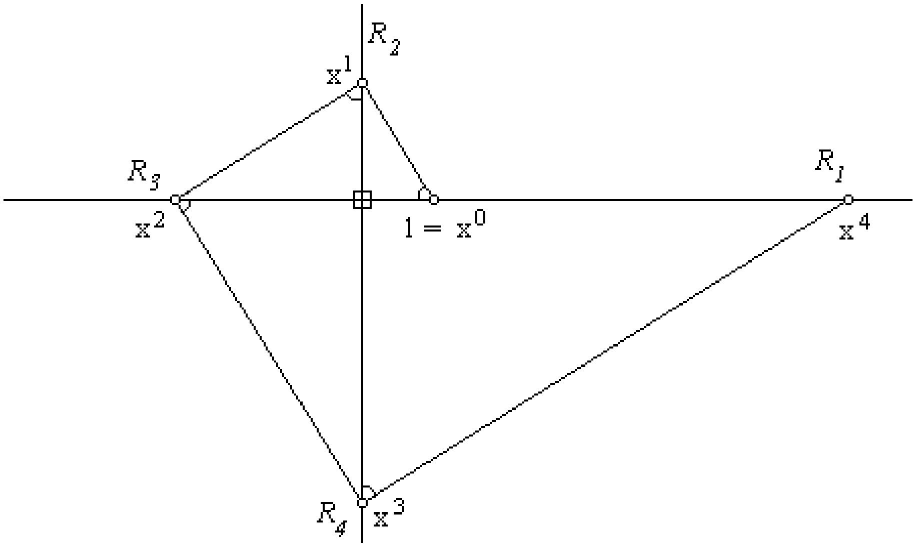


Diagram 2. Spiral yielding sequence of terminal edges:
$x^0, x^1, x^2, x^3, \ldots, x^n, \ldots$

Choosing a terminal edge, x^n, as a real number y, or $x^n = y$, or $x = y^{1/n}$ yields the sequence of terminal edges to be:

$y^{0/n},\ y^{1/n},\ y^{2/n},\ \ldots,\ y^{n/n},\ \ldots,\ y^{m/n},\ \ldots,$

which is depicted in diagram 3. Mathematical induction proves the preceding observations.

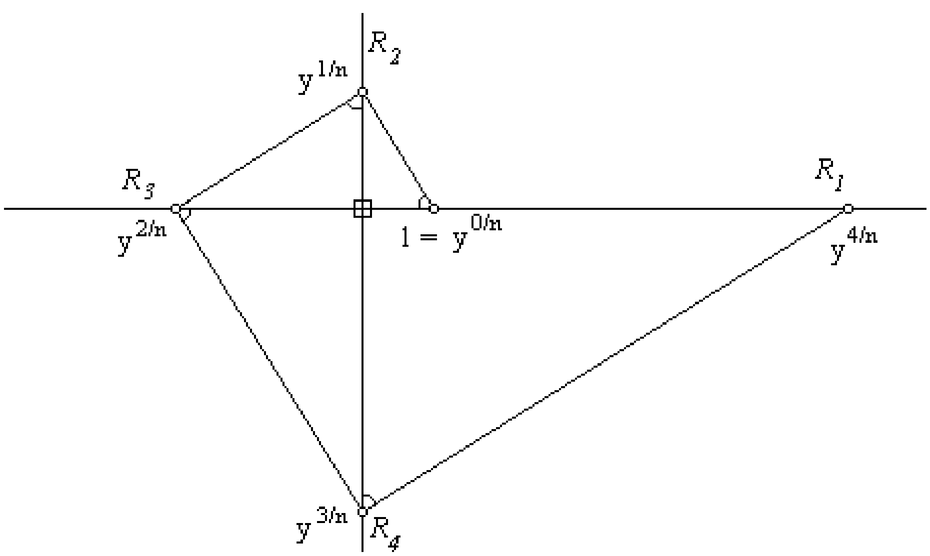


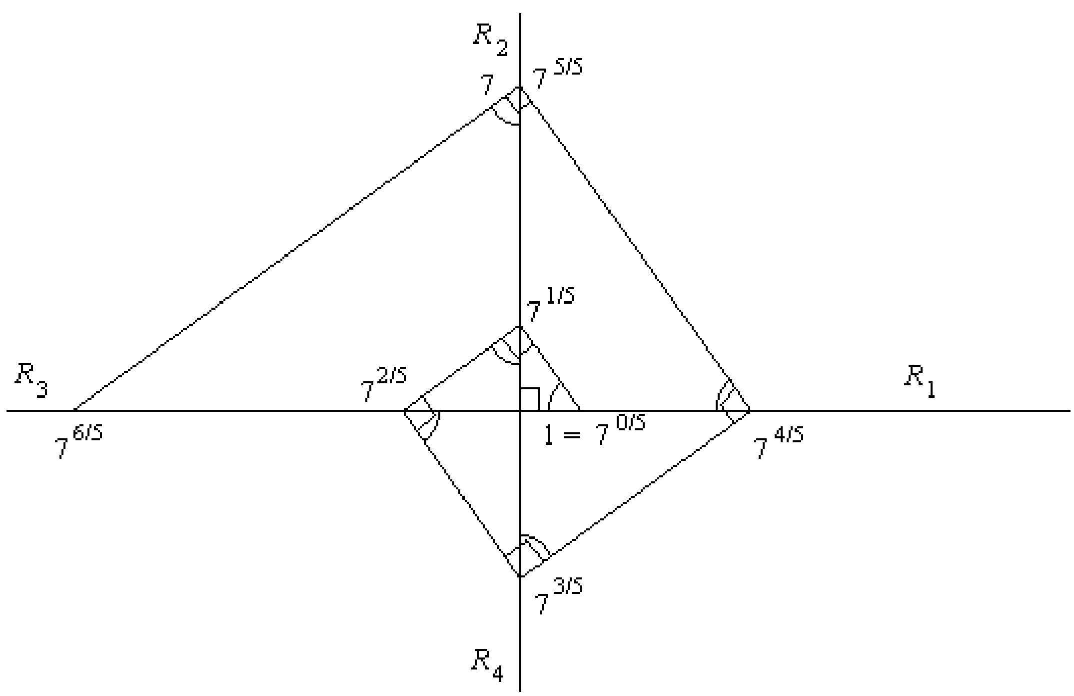
Diagram 3. Spiral yielding a sequence of terminal edges:
$y^{0/n},\ y^{1/n},\ y^{2/n},\ \ldots,\ y^{n/n},\ \ldots,\ y^{m/n},\ \ldots$

The following three examples, $7^{n/5}$, $7^{n/6}$, and $7^{n/7}$ with corresponding coordinates and calculator routines are examples of a combination of calculator routines and geometric models for helping students learn about $x^{m/n}$ whose learning style is complimentary to such a combination.

Example of $7^{n/5}$

Graphics System		Corresponding Calculator Result
	$7^{n/5}$	$C(7^{n/5})$: ON, 7, y^x, n, /, 5, = ___ nearest 10^{-3}
R_1	$7^{0/5}$	1
R_2	$7^{1/5}$	1.476
R_3	$7^{2/5}$	2.178
R_4	$7^{3/5}$	3.214
R_1	$7^{4/5}$	4.743
R_2	$7^{5/5}$	7
R_3	$7^{6/5}$	10.330

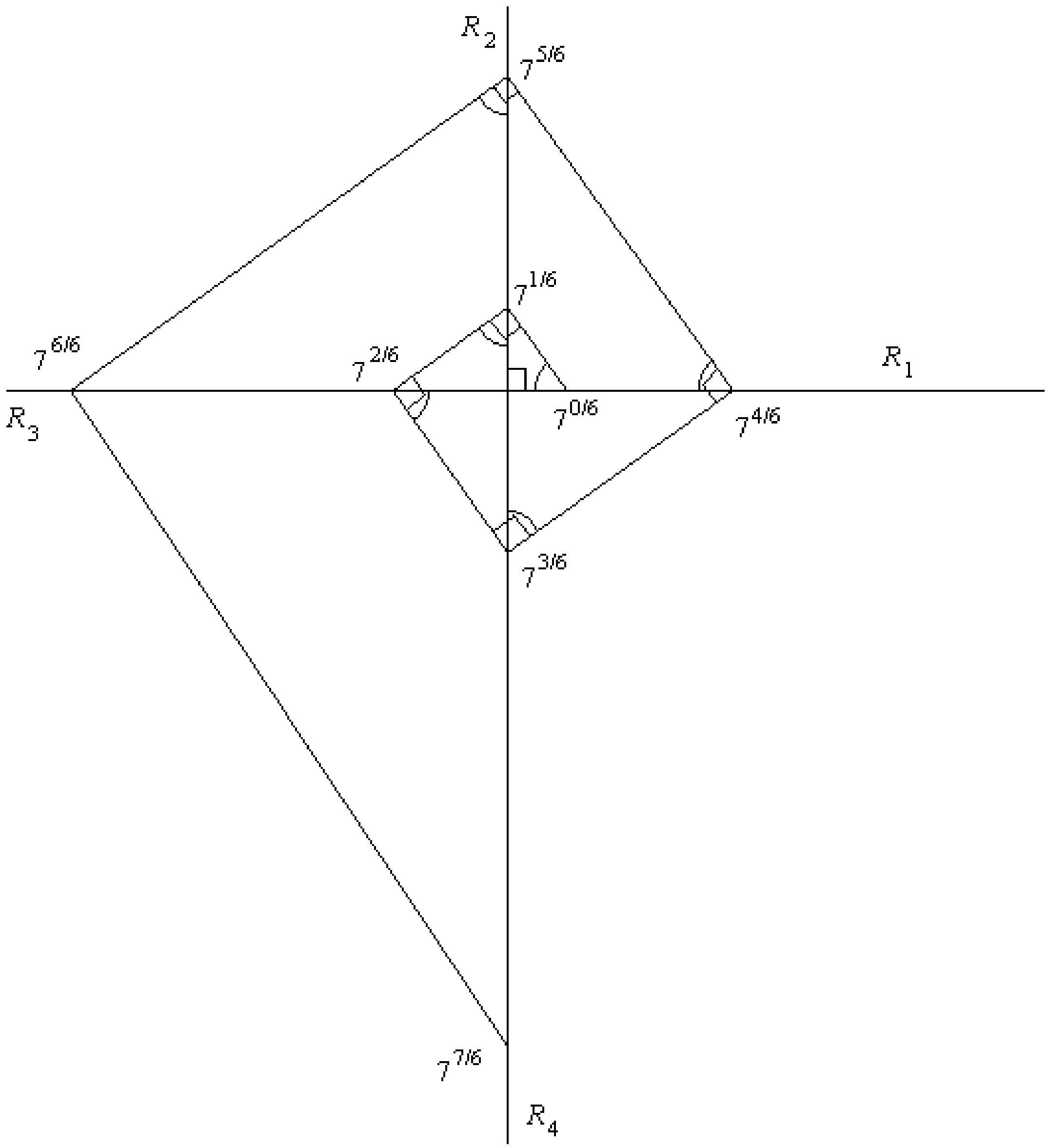
Geometric:



$7^{n/5}$ Spiral

Example of $7^{n/6}$

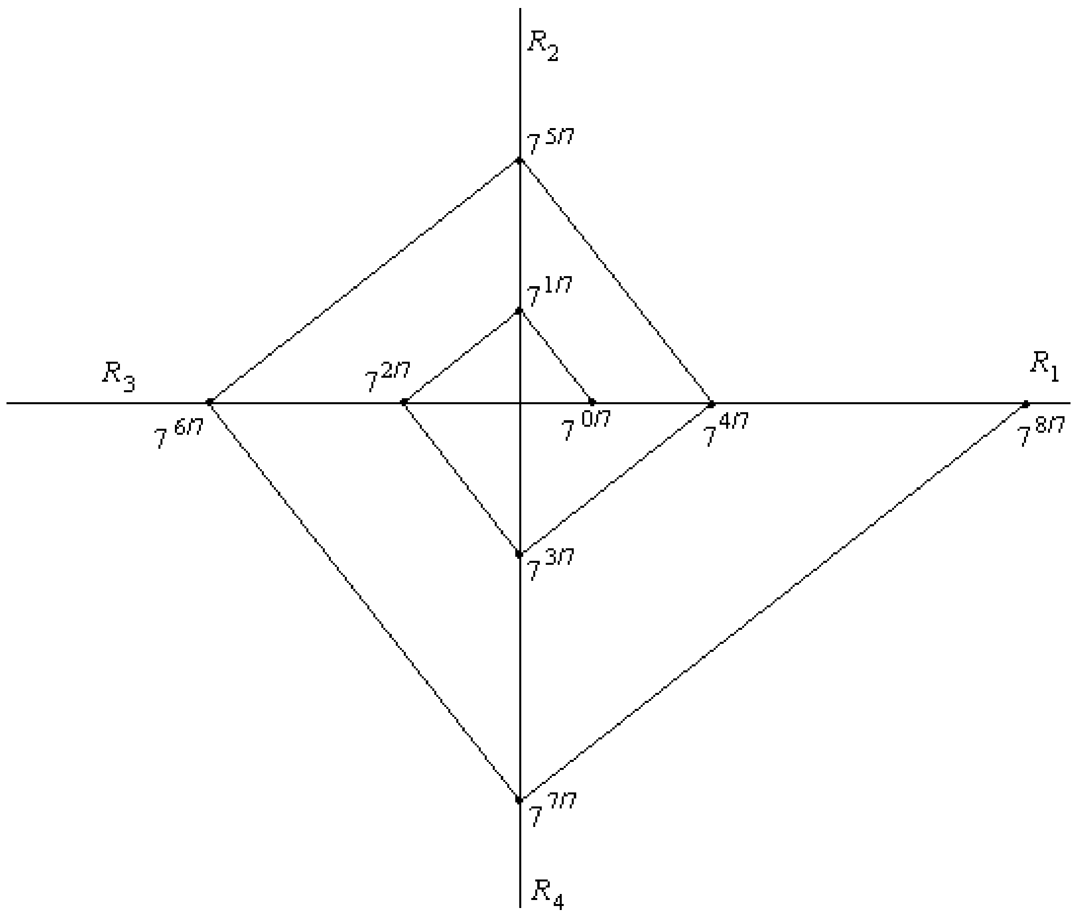
	Graphics System	Corresponding Calculator Result
	$7^{n/6}$	$C\left(7^{n/6}\right)$: ON, 7, y^x, n, /, 6, = ___ nearest 10^{-3}
R_1	$7^{0/6}$	1
R_2	$7^{1/6}$	1.383
R_3	$7^{2/6}$	1.913
R_4	$7^{3/6}$	2.646
R_1	$7^{4/6}$	3.659
R_2	$7^{5/6}$	5.061
R_3	$7^{6/6}$	7
R_4	$7^{7/6}$	9.682



R_2
$7^{5/6}$
$7^{1/6}$
$7^{6/6}$
$7^{2/6}$
R_1
R_3
$7^{0/6}$
$7^{4/6}$
$7^{3/6}$
$7^{7/6}$
R_4

Example of $7^{n/7}$

Graphics System		Corresponding Calculator Result
	$7^{n/7}$	$C\left(7^{n/7}\right)$: ON, 7, y^x, n, /, 7, = ___ nearest 10^{-3}
R_1	$7^{0/7}$	1
R_2	$7^{1/7}$	1.320
R_3	$7^{2/7}$	1.744
R_4	$7^{3/7}$	2.302
R_1	$7^{4/7}$	3.040
R_2	$7^{5/7}$	4.015
R_3	$7^{6/7}$	5.301
R_4	$7^{7/7}$	7
R_1	$7^{8/7}$	9.243



The author, throughout his professional career, has continually searched for models to enhance student conceptualization of numerical ideas, as well as other mathematical ideas. The author has been well aware of traditional techniques for modeling square roots. A couple of years ago, the author became aware of a sequence of similar trigons which generated a sequence of spirals which were useful to the ancient Greeks (about 300 BC) in designing war machines which required irrational measures. The author has reinterpreted the geometric situation to generate a sequence of similar right trigons (which seems to be more easily understood in our tradition) which in turn generates a sequence of spirals for representing $x^{m/n}$.