Sequences, Series and Convergence with the TI 92.

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Introduction.

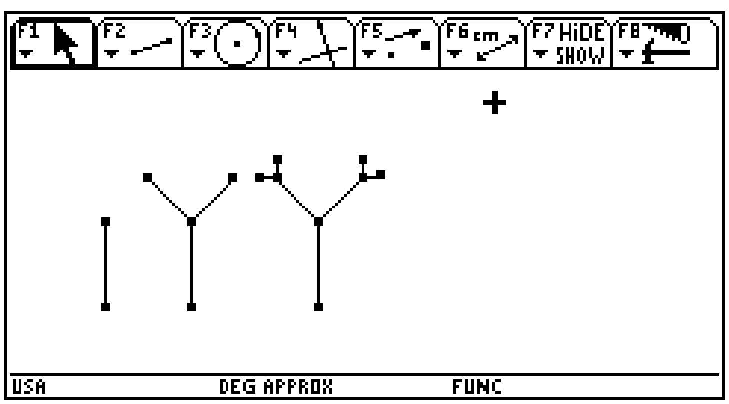
Students enrolling in calculus at Monash University, like many other calculus courses, are introduced to sequences and series in the same section of work. These students have difficulty distinguishing between a sequence and a series. These difficulties are compounded by the need for a sophisticated level of algebra as well as a sound knowledge of limits when dealing with convergence of a sequence or a series. The difficulties have been recognised by other authors including Morrel (1992)

The students completing the first semester of the first year calculus course at Monash University, again demonstrated a poor understanding of the difference between a sequence and a series as well as determining whether a series was convergent or not. In the second semester, a group of students including some who were repeating the subject, were provided with an investigative approach to the topic by utilising the multiple representations of the TI 92. The students were not only able to graph and tabulate sequences, partial sums of series, they were also able to compare algebraically derived limits for both sequences and series. The ability to use the symbolic algebra properties of the TI 92 further assisted the students in developing an understanding of the difference between a sequence and a series.

Student Activities.

A sample student activity follows:

The Geome Tree A demonstration using Cabri on the TI 92 was used to construct the sequence of Geome Trees.

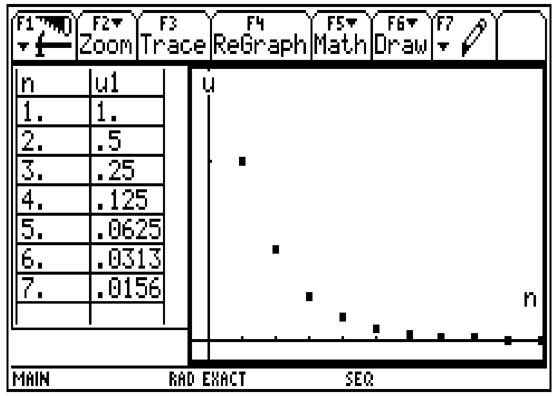


Students commenced by counting the number of segments. Most of the students were able to determine the sequence for the length of the new segments added, as well as write the formula for the n th term. The students were then asked to determine the partial sums for the length of a path, from the base to the end of a branch, before writing the series for the length of a path. Finally they were also required to determine the partial sums and the series for the number of segments.

Once the students had developed the sequence formula for the length of the branch and the series formula for the both the length of the path and the number of branches they were then able to plot the graphs using the sequence mode of the TI 92. A visual inspection of the tables and graphs allowed the students to see whether the series converged or diverged.

The lengths of the new segment added shown both by the table and the graph.

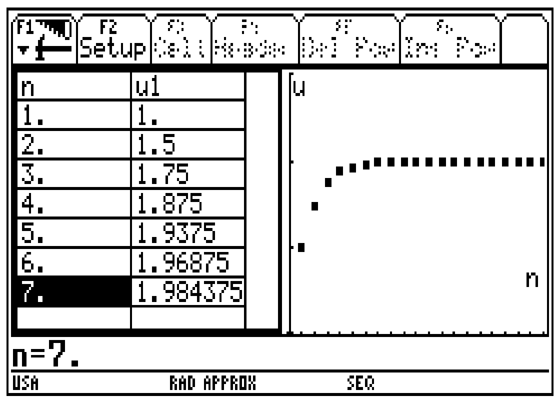
$$l_r = \frac{1}{2^{r-1}}$$



The partial sums for the length of the path where n represents the number of branches are also shown both by the table and the graph.

The formula for the partial sum is given by

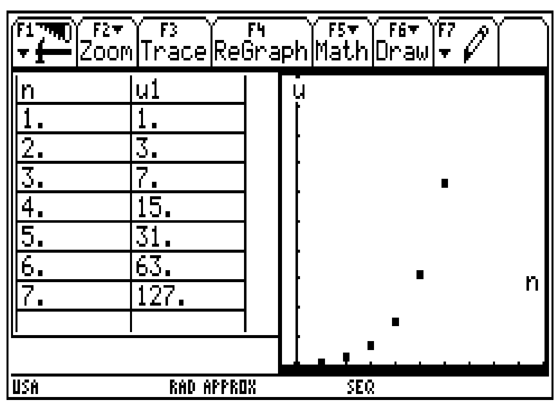
$$p_n = \sum_{r=1}^{n} \frac{1}{2^{r-1}}$$



The partial sums for the number of branches is also shown, by both the table and the graph.

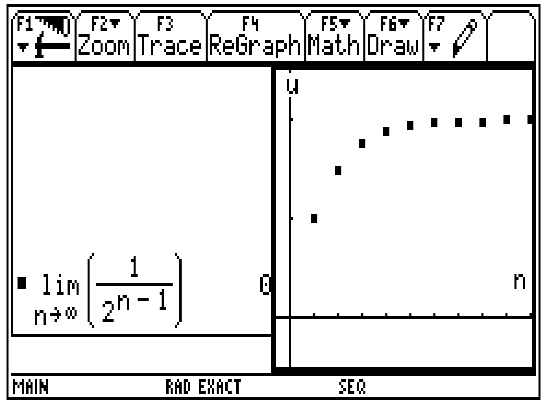
The formula for the partial sums is given by

$$b_n = \sum_{r=1}^{n} 2^{\,r-1}$$

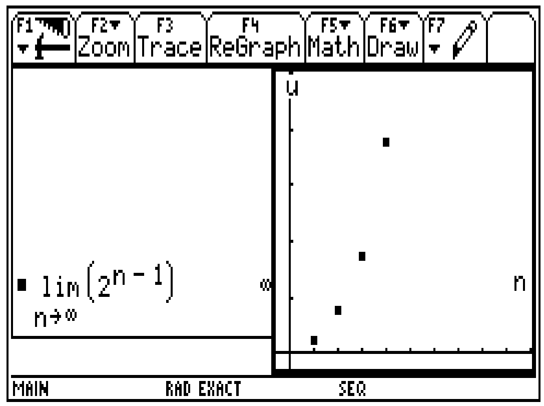


This intuitive introduction allowed the students to firstly develop an understanding of convergence and divergence and prepared them for an investigation involving the use of limits on the TI 92. By using the split screen mode of the TI 92 they were able to view both the graph of a sequence (including a sequence of partial sums) and a table of values for the terms of a sequence. The comparison of a number of examples enabled them to intuitively determine that the limit of the sequence of terms for a converging series was 0, while for a divergent series any value was possible including 0 and ∞ .

By comparing the graph of the partial sums of the length of the path with the limit of the sequence it was seen by the students that the series is converging and that the limit of the corresponding sequence of terms is zero.



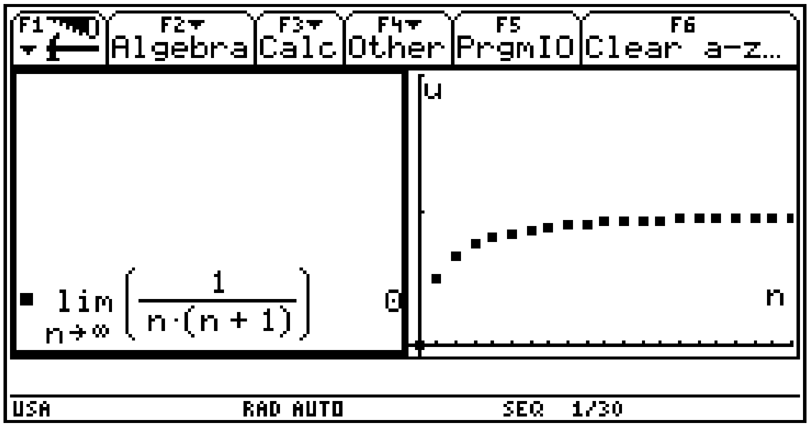
The graph of the partial sums of the total number of segments portrays the series as divergent and this is confirmed by the limit of the corresponding sequence of terms which is not equal to zero.



The use of contextual examples is important as it provided students with visual support to assist the development of their understanding of the terms sequence, partial sum and series, as well as experience with the summation sign. Having completed contextual examples related to the Koch Snowflake and the Sierpinska triangle they then moved onto examples involving other converging or diverging series such as the following

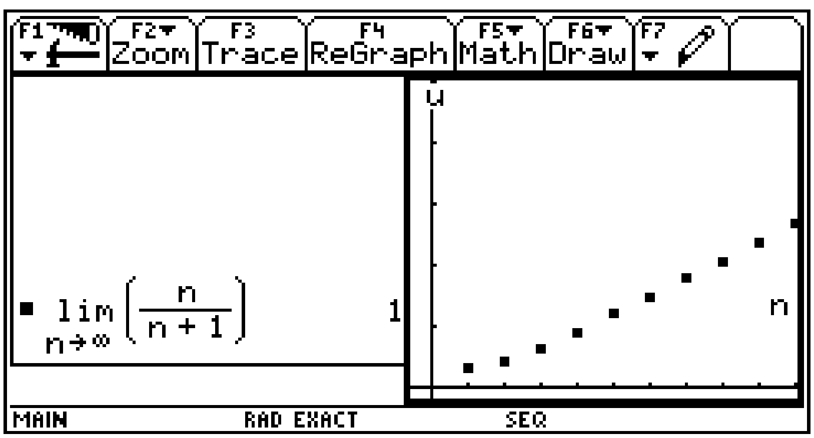
Example 1.

Consider the series $\sum_{n=1}^{\infty}\frac{1}{n(n+1)}$. Plot the sequence of partial sums and then determine whether the series converges or not.



Example 2.

Consider the series $\sum_{n=1}^{\infty} \frac{n}{n+1}$. Plot the sequence of partial sums and then determine whether the series converges or not. In this example it was important for the students to recognise that the sequence consisting of the terms of the series converges to 1 and therefore the series is divergent.



RESULTS

At the conclusion of the sessions on Sequences and Series the students were given a test covering the work completed. A portion of the questions were the same as on the test in first semester, while others were adapted to suit the different approach used in second semester. University policy determined that the students were only allowed to use scientific calculators in the test. The results for the comparative questions are shown in Table 1.

	First semester % correct $n = 43$	Second semester % correct $n = 28$
Question 1. Determine whether $\lim_{x \to 0} \frac{1}{x^2}$, exists or not. Calculate the limit if it exists.	23%	18%
Question 2. Determine whether the following sequence converges or not. If it converges find the value that it converges to. $\left\{1 - \frac{1}{n}\right\}_{n=1}^{\infty}$	*not applicable*	71%
Question 3. State whether the following series diverges or converges, giving reasons $\sum_{n=1}^{\infty} \frac{n-1}{n}$	23%	54%

Table 1. Comparative results from semester 1 and 2 for similar questions.

When comparing the performances from questions 2 and 3 in semester two, only 11 students or 39 % of the group were able to provide correct solutions to both questions, indicating that students were still having difficulty when comparing the convergence of a sequence and that of a series. The results in the table demonstrate that over half of those students tested were able to recognise and justify the correct solution for question 3 which is a marked increase on the first semesters result.

Students were also required to construct concept maps using at least the following terms: sequence, series, partial sums, limits, convergence and divergence. For many of the students this was the first time they had been asked to draw a concept map. These concept maps were scored according to the methods of Novak and Gowin (1984) and then the scores from the concept maps were compared with the students responses on the test. The students' performance on the test was reflected in their construction of the concept maps. Students lack of understanding of the difference between the convergence of a sequence and that of a series was evident. By considering the concept maps and the test results it appeared that the students' difficulty lay in the notion of finding the limit as n approached infinity for the convergence of a sequence, in contrast with the limit as n approached infinity for the terms of a series being zero when a series converges.

Conclusion

The effects of the work completed using both a contextual base and the Texas TI 92 has enhanced the performance of students in the first year calculus course at Monash University. Although the results from this alternative method of instruction are encouraging, further research will need to be undertaken to determine the ongoing effectiveness of these methods. Particularly by developing the students ability to recognise the uniqueness of the limit when considering the convergence or divergence of a series.

References

Morrel, J. H. (1992). A gentle introduction to infinite series using a graphing calculator. *Primus* 1992 Vol 2 (1).

Novak, J.D. & Gowin, D. B. (1984). *Learning how to learn.* Cambridge University Press: Cambridge.