# Using the TI-82 and TI-85 Graphing Calculators to Learn About Angles

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#### Abstract

The programming and graphing features of the TI-82 and TI-85 calculators provide an ideal tool for trigonometric laboratories. Programs which enable students to explore the properties of angles and circular arcs are utilized in an in class laboratory. The laboratory allows students to approximate  $\pi$ , discover the relation between radians and degrees, and to discover the formula for the length of a circular arc.

## 1 Introduction

This paper presents a trigonometry laboratory which uses the TI-82 and TI-85 graphing calculators. This laboratory is designed to be completed during one class period by students working in groups of two or three. The content of the laboratory is different from that usually found in published trigonometry laboratories, and it makes specific use of the TI-82 and TI-85. However, it could be adapted for classes using other graphing technologies.

The laboratory is designed to teach students about the relationship between radian measure and degree measure of angles. Students also approximate  $\pi$  to two decimal places and discover the relationship between the length of a circular arc and the radius of the circle containing the circular arc. The laboratory requires two programs, an introductory lecture, and a post laboratory lecture.

During the introductory lecture the instructor should define the following concepts: angle, vertex of an angle, angle in standard position, coterminal angles, central angle of a circle, sector of a circle, and circular arc of a sector of a circle. After this brief lecture students can proceed with the laboratory.

<sup>\*</sup>This research was carried out with support from a Southeastern Faculty Research Grant.

During the post laboratory lecture students should be provided with the definition of a radian and the relationship between radians and degrees. The number  $\pi$  should be discussed in relationship to its definition and the number P found in the laboratory. Finally the formula  $s = r\theta$  should be presented. In this formula s is the length of a circular arc, r is the radius of the circle containing the circular arc, and  $\theta$  is the central angle of the circle which subtends the circular arc.

The remainder of this paper provides the graphing calculator programs and the laboratory.

## 2 Angles Worksheet

#### Description of the ANGLES program for the TI-85.

Have your instructor transfer the **ANGLES** program to your calculator. Run the program **ANGLES** by entering the sequence PRGM <NAMES> <ANGLE> ENTER on your calculator. Your screen will read as shown below.

(Insert picture of screen here.)

Press either 0 or 1 and then press ENTER. Now input a value  $\theta$  for an angle. For demonstration purposes we will use 25° for  $\theta$ . Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows the initial and terminal sides of  $\theta$  in standard position. The initial side of  $\theta$  is the horizontal ray shown on the screen. Note that the screen does not show the *x*-axis or the *y*-axis. Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

To run **ANGLES** again press ENTER.

#### Description of the ANGLES program for the TI-82.

Have your instructor transfer the **ANGLES** program to your calculator. Run the program **ANGLES** by entering the sequence PRGM <ANGLES> ENTER on your calculator. Your screen will read as shown below.

(Insert picture of screen here.)

Press either 0 or 1 and then press ENTER. Now input a value  $\theta$  for an angle. For demonstration purposes we will use 25° for  $\theta$ . Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows the initial and terminal sides of  $\theta$  in standard position. The initial side of  $\theta$  is the horizontal ray shown on the screen. Note that the screen does not show the *x*-axis or the *y*-axis. Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows that the initial side of  $\theta$  had to be rotated 0.0694444444 rotations in the counterclockwise direction to obtain the terminal side of  $\theta$  when  $\theta = 25^{\circ}$ .

To run **ANGLES** again press ENTER.

#### **ANGLES** Questions

1. Assume that  $\theta$  is in standard position. Use **ANGLES** to fill in each of the tables given below.

$\theta$	$45^{\circ}$	$90^{\circ}$	$180^{\circ}$	$300^{\circ}$	$360^{\circ}$
Number of rota-					
tions to get from					
the initial to the					
terminal side.					
Direction of ro-					
tation (c or cc).					

θ	-90°	$-180^{\circ}$	-360°	720°	$-1170^{\circ}$
Number of rota-					
tions to get from					
the initial to the					
terminal side.					
Direction of ro-					
tation (c or cc).					

θ	1 radian	2 radians	3 radians	5 radians	7 radians
Number of rota-					
tions to get from					
the initial to the					
terminal side.					
Direction of ro-					
tation (c or cc).					

heta	-1 radians	-3 radians	-6 radians	-30 radians
Number of rota-				
tions to get from				
the initial to the				
terminal side.				
Direction of ro-				
tation (c or cc).				

2. If a positive angle is in standard position, then the initial side is rotated in

the \_\_\_\_\_\_ direction to get the terminal side.

- If a negative angle is in standard position, then the initial side is rotated in the \_\_\_\_\_\_ direction to get the terminal side.
- 4. In the next table you are to use **ANGLES** to determine if a given pair of angles are coterminal.

Angle pairs	$195^{\circ}$	$-165^{\circ}$	37°	$-323^{\circ}$	$117^{\circ}$	$478^{\circ}$	927°	$-1235^{\circ}$
Coterminal?								

5. The measure of the angle in standard position in which the initial side is rotated 0.5 rotations counterclockwise is \_\_\_\_\_\_ degrees.

6. Use **ANGLES** to answer the following question. Your answer should be accurate to two decimal places.

The measure of the angle in standard position in which the initial side is rotated 0.5 rotations counterclockwise is P =\_\_\_\_\_ radians.

- 7. Use 5 and 6 to fill in the following blanks.
  - 1 radian = \_\_\_\_\_ degrees (approximately)
  - 1 degree = \_\_\_\_\_ radians (approximately)

Use the tables from 1, your answer to 5, and your value of P from 6 to complete 8, 9, 10, and 11.

- 8. If θ is a positive angle in standard position measured in degrees, then the initial side of θ goes through \_\_\_\_\_\_ rotations in the \_\_\_\_\_\_ direction to obtain the terminal side.
- 9. If θ is a negative angle in standard position measured in degrees, then the initial side of θ goes through \_\_\_\_\_\_ rotations in the \_\_\_\_\_\_ direction to obtain the terminal side.
- 10. If  $\theta$  is a positive angle in standard position measured in radians, then the initial side of  $\theta$  goes through \_\_\_\_\_\_ rotations in the \_\_\_\_\_\_ direction to obtain the terminal side.
- 11. If  $\theta$  is a negative angle in standard position measured in radians, then the initial side of  $\theta$  goes through \_\_\_\_\_\_ rotations in the \_\_\_\_\_\_ direction to obtain the terminal side.

#### Description of the ARCS program for the TI-85.

Have your instructor transfer the **ARCS** program to your calculator. Run the program **ARCS** by entering the sequence PRGM <NAMES> <ARCS> ENTER on your calculator. Your screen will read as shown below.

(Insert picture of screen here.)

Press either 0 or 1 and then press ENTER. Now input a value  $\theta$  for an angle. For demonstration purposes we will use 25° for  $\theta$ . Press ENTER. Now input a value r for the radius. For demonstration purposes we will use r = 2. Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows a sector of a circle with radius 2 and central angle  $25^{\circ}$ . Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows that the length of the circular arc of the sector is 0.872664625997 when  $\theta = 25^{\circ}$  and r = 2.

To run **ARCS** again press |ENTER| |ENTER|.

#### Description of the ARCS program for the TI-82.

Have your instructor transfer the **ARCS** program to your calculator. Run the program **ARCS** by entering the sequence PRGM < ARCS > ENTER on your calculator. Your screen will read as shown below.

(Insert picture of screen here.)

Press either 0 or 1 and then press ENTER. Now input a value  $\theta$  for an angle. For demonstration purposes we will use 25° for  $\theta$ . Press

ENTER. Now input a value r for the radius. For demonstration purposes we will use r = 2. Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows a sector of a circle with radius 2 and central angle  $25^{\circ}$ . Press ENTER. Your screen will read as shown below.

(Insert picture of screen here.)

The above screen shows that the length of the circular arc of the sector is 0.872664626 when  $\theta = 25^{\circ}$  and r = 2.

To run **ARCS** again press ENTER ENTER.

#### **ARCS** Questions

12. Use **ARCS** to fill in each of the tables given below.

Central angle of	$27^{\circ}$	$115^{\circ}$	$212^{\circ}$	$300^{\circ}$	$360^{\circ}$
sector.					
Radius of sector.	1	2	3	1	2
Length of circu-					
lar arc of sector.					

Central angle of	1 radian	2.75 radians	3 radians	6.2 radians	0.5 radians
sector.					
Radius of sector.	1	1	1	1	1
Length of circu-					
lar arc of sector.					

Central angle of	1 radian	2.75  radians	3 radians	6.2 radians	0.5 radians
sector.					
Radius of sector.	2	2	2	2	2
Length of circu-					
lar arc of sector.					

Central angle of	1 radian	2.75  radians	$3  \mathrm{radians}$	6.2 radians	0.5 radians
sector.					
Radius of sector.	3	3	3	3	3
Length of circu-					
lar arc of sector.					

- 13. Let  $\theta$  be the central angle (measured in radians) of a sector of a circle with radius 1. Let s be the length of the circular arc of the sector. Using the second table of 12, find a relation between s and  $\theta$ .
- 14. Let  $\theta$  be the central angle (measured in radians) of a sector of a circle with radius 2. Let s be the length of the circular arc of the sector. Using the third table of 12, find a relation between s and  $\theta$ .
- 15. Let  $\theta$  be the central angle (measured in radians) of a sector of a circle with radius 3. Let s be the length of the circular arc of the sector. Using the fourth table of 12, find a relation between s and  $\theta$ .
- 16. Let  $\theta$  be the central angle (measured in radians) of a sector of a circle with radius r. Let s be the length of the circular arc of the sector. Using 13, 14, and 15 find a relation between s and  $\theta$ .
- 17. Let  $\theta$  be the central angle (measured in degrees) of a sector of a circle with radius r. Let s be the length of the circular arc of the sector. Look at the first table of 12. Do you see any relation between s and  $\theta$ ? If so, what is the relation?

## 3 Angles Worksheet Programs

### **ANGLES** Program

#### **TI-85**

:ClLCD :Normal :Float :Radian :Param :Dec :DrawLine :SimulG :GridOff :AxesOff :LabelOff :FnOff :Disp "INPUT 0 FOR" :Disp "DEGREES AND 1" :Disp "FOR RADIANS" :Input M :Disp "INPUT  $\theta$ " :Input THETA :If M==0 :Then : $\pi$ \*THETA/180  $\rightarrow$  T :Else :THETA  $\rightarrow$  T :End : $-2.5495601173 \rightarrow$  xMin :2.5495601173  $\rightarrow$  xMax : $-1.5 \rightarrow$  yMin :1.5  $\rightarrow$  yMax :0  $\rightarrow$  tMin :2.9  $\rightarrow$  tMax :0.1  $\rightarrow$  tStep :xt1=t :yt1=0 :xt2=t\*cos (T) :yt2=t\*sin (T) :DispG :Pause :ClLCD :Disp abs (T)/2 $\pi$ , "ROTATIONS" :If T<0 :Then :Disp "CLOCKWISE" :Else :Disp "COUNTERCLOCKWISE" :End :Disp "WHEN  $\theta$ =",THETA :If M==0 :Then :Disp "DEGREES" :Else :Disp "RADIANS"

#### **TI-82**

:ClrHome :Param :Float :Normal :Radian :FnOff :AxesOff :Connected :GridOff :Simul :FullScreen :LabelOff :Disp "INPUT 0 FOR" :Disp "DEGREES AND 1" :Disp "FOR RADIANS" :Input M :Disp "INPUT  $\theta$ " :Input A :If M=0 :Then : $\pi*A/180 \rightarrow Z$  :Else :A  $\rightarrow Z$  :End : $-2.5495601173 \rightarrow Xmin$  :2.5495601173  $\rightarrow Xmax$  : $-1.5 \rightarrow Ymin$  :1.5  $\rightarrow Ymax$ :0  $\rightarrow Tmin$  :2.9  $\rightarrow Tmax$  :0.1  $\rightarrow Tstep$  :"T"  $\rightarrow X_{1T}$  :"0"  $\rightarrow Y_{1T}$ :"T\*cos (Z)"  $\rightarrow X_{2T}$  :"T\*sin (Z)"  $\rightarrow X_{2T}$  :DispGraph :Pause :ClrHome :Disp abs (Z)/2 $\pi$ ,"ROTATIONS" :If Z<0 :Then :Disp "CLOCKWISE" :Else :Disp "COUNTERCLOCKWISE" :End :Disp "WHEN  $\theta$  =", A :If M=0 :Then :Disp "DEGREES" :Else :Disp "RADIANS"

## ARCS Program

#### **TI-85**

:ClLCD :Normal :Float :Radian :Param :Dec :DrawLine :SimulG :GridOff :AxesOff :LabelOff :FnOff :Disp "INPUT 0 FOR" :Disp "DEGREES AND 1" :Disp "FOR RADIANS" :Input M :Disp "INPUT  $\theta$ " :Input THETA :Disp "INPUT RADIUS" :Input RAD :If M==0 :Then : $\pi$ \*THETA/180  $\rightarrow$  T :Else :THETA  $\rightarrow$  T :End :-2.5495601173  $\rightarrow$  xMin :2.5495601173  $\rightarrow$  xMax :-1.5  $\rightarrow$  yMin :1.5  $\rightarrow$  yMax :0  $\rightarrow$  tMin :T  $\rightarrow$  tMax :0.1  $\rightarrow$  tStep :xt1=t/T :yt1=0 :xt2=(t/T)cos (T) :yt2=(t/T)sin (T) :xt3=cos t :yt3=sin t :DispG :Pause :ClLCD :Disp "ARC LENGTH=", RAD\*T :Disp "WHEN r=", RAD :Disp "AND  $\theta$  =", THETA :If M==0 :Then :Disp "DEGREES" :Else :Disp "RADIANS" :End :Pause

#### TI-82

:ClrHome :Param :Float :Normal :Radian :FnOff :AxesOff :Connected :GridOff :Simul :LabelOff :FullScreen :Disp "INPUT 0 FOR" :Disp "DEGREES AND 1" :Disp "FOR RADIANS" :Input M :Disp "INPUT  $\theta$ " :Input A :Disp "INPUT RADIUS" :Input R :If M=0 :Then : $\pi*A/180 \rightarrow Z$  :Else :A  $\rightarrow Z$  :End :-2.549560117  $\rightarrow$  Xmin :2.549560117  $\rightarrow$  Xmax :-1.681624758  $\rightarrow$  Ymin :1.681624758  $\rightarrow$  Ymax :0  $\rightarrow$  Tmin :Z  $\rightarrow$  Tmax :0.1  $\rightarrow$  Tstep :"T/Z"  $\rightarrow$  X<sub>1T</sub> :"0"  $\rightarrow$  Y<sub>1T</sub> :"(T/Z)cos (Z)"  $\rightarrow$  X<sub>2T</sub> :"(T/Z)sin (Z)"  $\rightarrow$  Y<sub>2T</sub> :"cos T"  $\rightarrow$  X<sub>3T</sub> :"sin T"  $\rightarrow$  Y<sub>3T</sub> :DispGraph :Pause :ClrHome :Disp "ARC LENGTH=", R\*Z :Disp "WHEN R=", R :Disp "AND  $\theta$ =", A :If M=0 :Then :Disp "DEGREES" :Else :Disp "RADIANS" :End :Pause