Mathematical Analysis on the HP-48

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Abstract

The HP-48GX is used to investigate features of a function of n variables arising in the context of a nonlinear optimization problem in operations research. Interestingly, the function involves both discrete and continuous variables. The optimal solution requires not only determination of the values of the continuous variables but, more interestingly, also the number of variables per se to include. Special attention is directed to the investigation of the domain and monotonic properties of the function.

The technology supporting this presentation is the HP-48GX, referred to as a handheld computer, as a supercalculator, or (and seemingly least appropriately) as a calculator. Various other, but necessarily judiciously chosen, forms of supporting technology could satisfy our present computational demands. Our intention is not to make the case for one form of computational device to be used over another. In the present context the HP-48 meets our needs acceptably well.

Our intention is to demonstrate how technology, such as the HP-48, might assist a mathematical investigator/researcher in carrying out certain mathematical analyses.

We deal with a real-valued nonlinear function of several variables, having the form:

$$(i) \qquad f(N, x_1, x_2, ..., x_N)$$

Here, N is a positive integer and the x's represent probability values between 0 and 1. The particular version of this function with which we deal arises in the theory of detection or searching procedures. Perhaps the most appropriate general field to which this function belongs, indeed with respect to its application orientation, is operations research.

The context in which we describe the use of this function is as follows. Suppose there are finitely many different ways in which to carry out a successful (feasible) search. Each leads (it turns out) to a value of the integer N in the function f above, and then, accordingly, to N variables, namely the subscripted x's, and their properly assigned values. For example, there may be three feasible searches where one involves 11 variables, another 12, and the last 13. Then N = 11, 12, and 13. Of the finitely many different successful/feasible searches, we ask which is optimal? Here, "optimal" will mean "maximizes the probability of acquisition at all times during the search and the least amount of time is consumed in performing the search" (see [1]).

Omitting further specific contextual or circumstantial details, the aspect of the searching procedure we seek in the mathematical model with which we deal is the value of N that will make A least, where

$$(ii) \qquad A = -2\Pi\sigma_x\sigma_y \Sigma_{k=1}^{N} \ln[1 - x_k]$$

Or, equivalently (see [1])

$$(iii) \qquad A = 2\Pi N\sigma_x\sigma_y \ln \frac{NP(D)\,[1 - P(D)]^{(N-1)/2}}{1 - P(A) - [1 - P(D)]^N}$$

In the above, P(D) represents the probability of detection on one "look" or "pass", and P(A) represents the desired probability of acquisition, i.e., actually locating the object for which the search is conducted. Finally, A represents the total area to be searched, where searching a given region often (more than once) counts the revisited area (in the total area searched) as often. Of the different acceptable (feasible) choices of N, our focus is to find the N that will make A least. In other words, of the feasible solutions, we seek the optimal one, i.e., the optimal feasible solution (to use popular operations research language).

Notice in (ii) that A is a function of N variables, the x-values. In (iii) A, conveniently re-expressed, depends on the choice of P(D), P(A), and N. The reader should know that the possible values for N are predetermined by P(D) and P(A).

After some thought the investigator might come to conjecture, as this one did, that the value of N for which the feasible solution is optimal is the largest value of N for which there is a feasible solution. (Let us interject that this paper is not about how to find any one feasible solution. For that see [1] and [2].) And the use of a computational device can greatly assist the researcher in arriving at this conjecture, the main point of this paper. In fact, a computational or effective graphical device might suggest to the investigator that A is a monotonically decreasing function of N. We note then that the optimal solution will be the largest value of N for which there is a feasible solution (for the search is completed most rapidly if the total area, A, to be searched is smallest). The author has, in fact, proved that the largest N is optimal in the case of restricted domains of f, namely when P(D) > 3/4 and P(A) is unrestricted. See [2].

It appears that A is monotonically decreasing for a wide range of other P(D) values as well.

To see why one might want to turn to a computational device to assist in the analysis of the function expressed by (iii), the reader might try to show by hand methods, thinking of N as a continuous variable, that the derivative of A with respect to N is negative over the feasible values for N. In this context think of P(D) and P(A) as constant probability values, i.e., and more precisely, think of them as arbitrary but fixed probability values. The expression for the derivative is, one is likely to conclude, complicated and difficult to analyze. If the reader has an HP-48, have the HP compute the derivative. For reader convenience, the derivative is shown in HP syntax on the back pages of this paper.

One might numerically investigate the behavior of the derivative of A with respect to N using a Monte Carlo approach. Special interest is in order for the case when values of P(D), assigned randomly, are less than or equal to 3/4, and where arbitrary but fixed probability values of P(D) and P(A) are assigned, again, at random. To accomplish the latter, one might create a program that generates random input values and then evaluates the derivative of A. If the derivative is negative for random values of the variables--of course, N must be restricted to an appropriate set of values (see [2])--one might conjecture the derivative is always negative. Of course, this would not constitute a proof that the derivative is always negative--that A is a monotonically decreasing function of N for fixed P(D) and P(A)--but it does provide strong evidence for making that conjecture.

Another approach leading one to conjecture that A is decreasing as a function of N--this time thinking of N as a discrete, integer-valued variable--would be achieved by using expression (iii) above and forming the ratio obtained by setting N = n + 1 divided by the expression obtained when N = n. Should the ratio below

$$(iv) \qquad \frac{b_{n+1}}{b_n} = \frac{n+1}{n}\sqrt{q}\,\frac{1 - P(A) - q^n}{1 - P(A)\ q^{n+1}}$$

always be less than 1 for all appropriate choices of n and fixed values of P(A) and P(D), then again one would

be led to conjecture that A is a decreasing function of N for the fixed P(A) and P(D) employed, i.e., the optimal feasible solution appears to be obtained for the largest feasible N. See [2] to determine bounds for N which, as has been said, depend upon the input values of P(D) and P(A), i.e., see [2] to determine the appropriate domain values of N.

[1] Jarvinen, R.D. (1982). Conditions for an Optimal Search. In: Selected Studies: Physics-Astrophysics, Mathematics, History of Mathematics (T.M. Rassias and G.M. Rassias, ed.), pp. 93-98. North-Holland, Amsterdam.

[2] Jarvinen, R.D. (1991). Bounds for an Optimal Search. In: Constantin Caratheodory: An International Tribute (T.M. Rassias, ed.), Vol. I, pp. 543-547. World Scientific, London.