PARADIGM LOST:
A MODERN APPROACH TO TEACHING ORDINARY DIFFERENTIAL EQUATIONS

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* Differential Equations is a mid-range course in the mathematics curriculum at IBC.

* It's a "transition" course from lower level "methods" courses to upper level "theoretical" courses.

* As such, it is the first course that most students take in which theoretical mathematics is given a serious treatment.

* Differential Equations is a nice blend of theoretical(pure) mathematics and practical(applied) mathematics.

* Therefore, Diff. Eqns. is a "good" course!

TEACHING PHILOSOPHY

Strike a balance between the theoretical and applied nature of the material so that students

* are knowledgeable about techniques of obtaining a solution

* have at least a rudimentary knowledge of the underlying theory.

TWO MAIN OBJECTIVES

(1) To impart a knowledge and understanding of the course content.

* techniques used to obtain solutions for various classes of differential equations.
  * teach students how to analyze a differential equation in order to make a qualitative statement about the solution.

(2) A broader goal is to encourage growth in each student toward the next level of mathematical thinking.

* modeling a physical phenomenon mathematically
* analyzing a model
* bringing theoretical questions to bear on a model
* using theorems to answer theoretical questions

* thinking more abstractly about mathematics

THE OLD PARADIGM
(How I was taught)

* Differential Equations has traditionally been taught as a `methods' course. Different types of differential equations are discussed along with the method of obtaining their solution.

* The student is then left to work through the concomitant details of this new method, which can involve lengthy algebraic manipulations, a healthy dose of differential calculus and of course, the correct execution of the new technique.
(e.g. Variation of Parameters)

* All in all this can be a daunting task fraught with peril for the unwary student. Consequently, it does little to pique the interest or curiosity of most students.

THE NEW PARADIGM
(How I teach)

The new paradigm emphasizes:
* the modeling process
* analysis of the differential equation
* *understanding* the qualitative behavior of the solution
* use of computer technology to visualize and thus---I hope---understand

* `talking' about mathematics(=writing), not just `doing' mathematics(=procedures/methods)

EFFECTING THE PARADIGM SHIFT

Here's what I do in Diff. Eqns. to effect the paradigm shift:

(1) Introduce students to "chaos". This is a contemporary topic that the students find fascinating in its own right. However, it has the added benefit of debunking the myth that mathematics is a completed subject, rather than one that is constantly growing and revitalizing itself.

(2) Use different media to communicate the material.
- video on "Chaos & Fractals" by Robert Devaney
- in class computer demonstrations (just a few)
- computer laboratory experiences (about 5 or 6)
- video on "Transition to Chaos" by Robert Devaney (Friday afternoon *with enticement*---pizza and pop!)
- in class lectures(not ready to give these up yet!)

(3) Give the class a project ("real life" problem that arose in the Chemistry Dept. at IBC).

(4) Encourage students to talk at a local Student Symposium. Many of them take me up on it. Topics are typically in the area of Chaos or Nonlinear Dynamics.

(5) Mandatory computer lab component.(MODELS software package is used.) British proverb: "*What one can not clearly state, one does not know*!" Math is no exception.

WHY BOTHER USING COMPUTER TECHNOLOGY?

* Some people view the use of computers in math classes as the *panacea* that we've all been waiting for.

* In fact, it has been my experience that unless a piece of computer software is used properly, it amounts to nothing more than a high-tech pedagogical *placebo*.

* I view computers as a means to an end and not an end in themselves.

* The objective is to impart knowledge and understanding and to develop critical thinking skills in mathematics.

* To the degree that computers aid in accomplishing this, they are a useful tool.

* The computers do the calculations; the students still have to do the *thinking*!

WHAT DOES A COMPUTER LAB COMPONENT DO FOR THE STUDENTS?

* Ensures a minimal level of computer competency.

* Enables students to focus on mathematical concepts rather than tedious arithmetic or lengthy algebraic manipulations

* Facilitates <u>exploring</u> & <u>discovering,</u> which in turn encourages students to ask questions. What if...

* Students are more likely to think abstractly and critically because they can <u>focus on the concepts</u>. (Computer is doing all the calculations.)

* Aids in visualization. ("Brings the material to life!"[former student])

* Improves their writing skills.(Students are required to write lab reports---in complete sentences---with a verb. This is how we reinforce "writing across the curriculum" in the math dept.)

* The students enjoy learning about differential equations and some even end up `liking' it!

THE BAD NEWS

Implementing software into a course entails a prodigious amount of work.

* A 12 page guide explaining how the software works.

* System specific items(login, logout, printing, ...)

* Take class into lab and walk them through basic commands

* Compose meaningful lab experiments, aaaargh! (This is getting a bit easier with C●ODE●E and other resources.)

* Grade lab experiments--- ⌚ tick, tick, tick ⌚

* Improve lab experiments for next time and come up with new ones---no resting on laurels!

* All in all, this represents a considerable time commitment.

THE GOOD NEWS

* Students find the course more interesting.(Some cited the labs as the most exciting part of the course.)

* In general, they like the computer lab experience.(It takes time, but they like the results.)

* They are more confident and go away feeling good about diff. eqns.

* Students go away with a <u>better understanding</u> of the material.

PROOF POSITIVE

* MODELS was a great asset to the class. It was very easy to use and really helped to visualize what was going on.

* All in all I thought MODELS was great, it helped bring a lot of the equations and things to life for me.

* I liked MODELS more than I thought I would. After you get the hang of it, it's very easy to work with and produces excellent graphs.

* This[MODELS] was the bright spot for me in this class. It is my opinion that MODELS is a fantastic asset to this class.

* I liked the material of the course in general, but it was especially interesting when presented through MODELS labs.

* This was the best math course I have ever taken here ... The labs were fun and helped me to visualize what was going on. I loved the class!

* The instructor is interesting... Having been very "afraid"[of diff. eqns.], I now feel comfortable and more confident.

WHAT'S SO HARD ABOUT WRITING A <u>GOOD</u> LAB?

* Must be interesting(not just something to keep the students occupied)

* Instructions must be clear and easy to follow.(=sailor proof!)

* Must be an element of <u>exploration</u> & <u>discovery</u> as opposed to just a `list of things to do'.(=cookbook)

* Coming up with meaningful questions to ask.(Should be doable but not trivial.)

* Most of the time, you must create a boilerplate program and load it onto the network(or computers as the case may be) so students don't have to concern themselves with things like:
   ☹ an inordinate amount of data entry
   ☹ getting all the bells and whistles set up right
   ☹ discretization of the o.d.e.(s)

* Challenging yet reasonable.

* Provide opportunity to experiment(=play) by asking questions that have many correct answers. (Find a function such that ...)

LIST OF LABS

1. Microbe Mania(Modeling: Discrete vs. Continuous)

2. Harvesting a Species(Phase plane & stability analysis)

3. Cobwebs & Chaos(Understanding logistics equation globally)

4. Predators & Prey(Investigating an autonomous system)

5. Eigenvalues & Eigenvectors(System of first order linear o.d.e.s)

6. Falling Bodies(Bodies in free-fall: linear drag vs. Newtonian drag.)

7. Chase Strategies in Football(a <u>tractable</u> 2nd order, nonlinear o.d.e.)

TOPICS COVERED

* Linear o.d.e.s

* Separable equations; Exact equations; Nonlinear vs. linear

* Almost Linear Systems; Autonomous Systems/Stability

* Falling body problem

* Undetermined coefficients; Euler equations

* Population dynamics

* Laplace Transform method(5 days)

* 2nd order, homogeneous, constant coefficient

* Fundamental Solutions; L. I. ; Wronskian

* Complex roots; repeated roots; reduction of order

* Power series; series solution near ordinary point

* Eigenvalues/eigenvectors; Complex eigenvalues

* Applications of 1st order o.d.e.s (mixing problems & time of death)

* Phase plane: Linear Systems

* 8 days(= 2weeks) are spent in the lab