Examples from Calculus with Scientific Workplace

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Abstract In this paper, we shall use the software, Scientific Workplace, to demonstrate how we can interact with functions which are defined in Maple. We also use examples from Calculus to illustrate how we can teach live mathematics and communicate with others more effectively and efficiently without learning the syntax.

1. Introduction

There are many text books written about the use of certain computer algebra system (CAS) in teaching certain course. But we have to admit that if we don't know certain CAS syntax, then we could not grasp the importance of what authors want to convey. As a result, readers often skip sections which are demonstrated with the CAS program.

In this paper, we use the following examples to illustrate that we can teach mathematics and communicate with others more efficiently and effectively with Scientific Workplace.

2. Link a Pre-defined function in Maple

We first demonstrate how we can link a pre-defined function in Maple with a user-defined function in Scientific Workplace. As a result, users can experiment such function directly inside Scientific Workplace.

2.1. In Maple

When we ask help on the function **interp**, we get the following:

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PARAMETERS: x - list or vector of independent values, x[1],..x[n+1]
y - list or vector of dependent values, y[1],..y[n+1] v - variable
to be used in polynomial.
SYNOPSIS: The function interp computes the polynomial of degree
less than or equal to n in the variable v which interpolates the points
(x[1],y[1]), (x[2],y[2]), ..., (x[n+1],y[n+1]).  - If the same x-value
is entered twice, it is an error, whether or not the same y-value
is entered; all independent values must be distinct.
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EXAMPLE:
> interp([0,1,2,3], [0,3,1,3], z);
3/2 z^3 - 7 z^2 + 17/2 z.
```

2.2. In Scientific Workplace

Let's define a corresponding Scientific Word name by I. Thus we obtain (by using **Evaluate**)

$I([0,1,2,3],[0,3,1,3],z)) = \frac{3}{2}z^3 - 7z^2 + \frac{17}{2}z$

Example 1. *Compare the function* $f(x) = \frac{x}{x^2+1}$ *over the interval* $[0,4]$, *with different interpolating polynomials.*

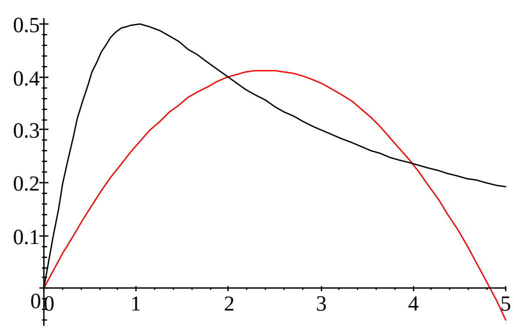
Step 1: We define the function f and next we obtain the interpolated polynomial of degree 2 as follows:

$I([0,2,4],[f(0),f(2),f(4)],t) = -\frac{6}{85}t^2 + \frac{29}{85}t.$

Step2: We define $L_2(t)$ to be the interpolated polynomial and plot f and L_2 together.

$L_2(t) = -\frac{6}{85}t^2 + \frac{29}{85}t$

f, L_2

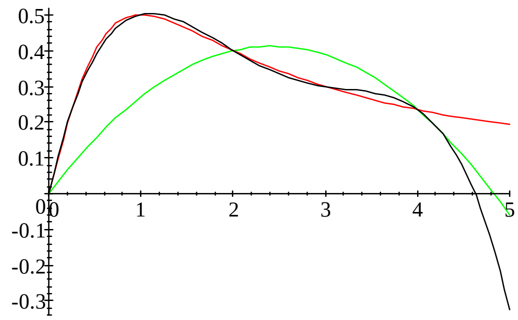


Step 3: We repeat the process in Step 2 with a fourth-degree polynomial.

$I([0,1,2,3,4],[f(0),f(1),f(2),f(3),f(4)],t) = -\frac{2}{85}t^4 + \frac{41}{170}t^3 - \frac{73}{85}t^2$
$+\frac{97}{85}t$

We define $L_4(t) = -\frac{2}{85}t^4 + \frac{41}{170}t^3 - \frac{73}{85}t^2 + \frac{97}{85}t$, and plot f, L_2, and L_4 all together as follows:

f, L_2, L_4



Question: Will Lagrange interpolation always work and work well? The answer can be seen from the following two theorems.

Theorem 2. *For every family of knots there is a function that will be badly represented by the largrange interpolating polynomials.*

Theorem 3. *For every function there is a bad choice of knots.*

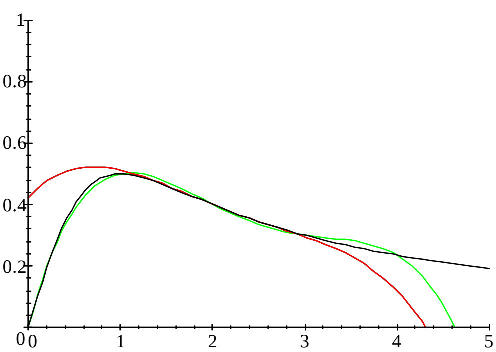
2.3. Taylor Expansion of f.

By choosing series expansion of f with degree 5, and in the power of $x-2$ yields

$f(x) = \frac{2}{5} - \frac{3}{25}(x-2) + \frac{2}{125}(x-2)^2 + \frac{7}{625}(x-2)^3 - \frac{38}{3125}(x-2)^4$
$+O\left((x-2)^5\right).$

Let's set $T_4(x) = \frac{2}{5} - \frac{3}{25}(x-2) + \frac{2}{125}(x-2)^2 + \frac{7}{625}(x-2)^3 - \frac{38}{3125}(x-2)^4.$

f, L_4, T_4



Remark 1. *The Taylor polynomial T_4 of f is a good estimate for the function f in the interval $[0,4]$ since we choose the expansion to be around $x=2$. Which method will give a"better" approximation to the function f ? Lagrange or Taylor polynomial?*

3. Defining a function with several variables inside Scientific Workplace

We shall approximate an integral numerically by using Riemann Sum midpoint rule and right point rule. From this experiment, we hope students will know that given a function, to estimate its numerical integration, they can experiment different quadratures (to achieve answers) and finally determine which method is the best for this given function.

To define a function with several variables in Maple or Mathematica may not be trivial to non-expert. But if we write

$$M_f(a,b,n) = \frac{b-a}{n}\sum_{i=0}^{n} f(a + \frac{b-a}{2n} + i\frac{b-a}{n}),$$

whoever knows Riemann sums would figure out that $M_f(a,b,n)$ is a function depends on a, b, and n, and represents a midpoint rule. Furthermore the function $M_f(a,b,n)$ **is ready for computation within Scientific Workplace**.

Example 4. *Approximate the following integral*

$$\int_0^1 \frac{(\sin x)\ln x}{x} dx.$$

(By using evaluate numerically from SW, we obtain $-.9818108$.)

3.1. Middle Boxes

If

$$f(x) = \begin{cases} \frac{(\sin x)\ln x}{x} & if \quad x \neq 0. \\ 0 & if \quad x = 0. \end{cases}$$

We define the midpoint rule as follows:

$$M_f(a,b,n) = \frac{b-a}{n}\sum_{i=0}^{n} f(a + \frac{b-a}{2n} + i\frac{b-a}{n}).$$

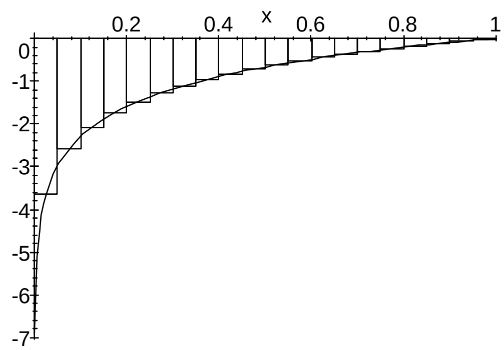
For computation, we define the function $M_f(a,b,n)$ by choosing **New definition** from Maple and choose **Evaluate** from Maple, we obtain

$$M_f(0,1,50) = -.9747265,$$

and

$$M_f(0,1,80) = -.9774187.$$

We could increase the number of evaluation n and make conjecture in determining if the integral exists. We note that we also can make a middle boxes plot (or left boxes, or right boxes plot) by choosing the **Calculus** as follows**.** *PlotApprox.Integral*.



3.2. Right Boxes:

We similarly experiment the right point rule by defining

$$R_f(a,b,n) = \frac{b-a}{n}\sum_{i=1}^{n} f(a + i\frac{b-a}{n}),$$

and obtain

$$R_f(0,1,50) = -.9242838,$$

and

$$R_f(0,1,80) = -.9429255.$$

Remark 2. *Students observe that the midpoint rule is more efficient in this case, but can they explain why?*