Some Examples on Using Maple to Increase Students' Understanding of Calculus

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1. Introduction

It has been almost a decade since Lean and Lively Calculus sent us down the road to calculus reform. There are still very great obstacles ahead. The first obstacle is that reforming calculus requires more faculty time. Secondly, most calculus reform efforts cost more money. Thirdly, teaching reform calculus requires more talent and training. The purpose of this article is to contribute our effort to overcome the above obstacles.

Through our experience during the reform of calculus, we saw an emphasis on classroom activities of four types: machine computation; cooperative learning, in which students work together; investigative learning; and writing. Therefore, we have tried to concentrate on the above areas by using the Computer Algebra System Maple as our tool.

2. Some Examples

Most of the algebraic manipulations featured in the traditional course can now be easily delegated to a computer. Using Maple, most of routine problems can be solved by one or two commands. Therefore, we should take advantage of the computer algebra system to train students to explore, conceptualize, and develop problem solving skills. The following are several examples we have used in our teaching for the above purpose:

Example 1. Graph f(x) = ln(sin(x)*cos(x)) and
g(x) = ln(sin(x)) + ln(cos(x)). What is the
relationship between the graphs? Does it
contradict the property
ln(x*y) = ln(x) + ln(y)?

To emphasize the logarithmic properties, when we were teaching the natural logarithm function, the exponential function and other exponential and logarithmic functions, instead of using typical exercises like condensing or expanding logarithmic expressions or even those artificial word problems, we assigned the above problem to our students.

Through the graphs of these two functions along with the graphs of sin(x) and cos(x), the students worked together to investigate the properties of sin, cos, and ln , and they had to show their answer clearly in writing.

Students can use Maple to draw the two graphs easily, but the two graphs looked entirely different. Does this contradict the property ln(x*y) = ln(x) + ln(y)? How would students explain it?

After several repetitions of the experiment and recalling the definitions and properties of ln, sin and cos, the students may observe that because of the periodic propertity of sin(x) and cos(x), the two graphs look different, whereas they are identical as long as ln(sin(x)cos(x)) and ln(sin(x))+ln(cos(x)) are defined.

Example 2. Evaluate the integral

$$\int_0^1 \sec\left(\frac{1}{4}\,\pi\,t\right) - \tan\left(\frac{1}{4}\,\pi\,t\right)\,dt$$

and graph the region whose area is given

by the integral.

Usually, the notion of tan or sec will give students who are majoring in liberal arts and reluctantly taking math such uneasiness that they may get completly stuck. Fortunately, by using Maple as their tool, students can easily find answers without any difficulty and at the same time gain back some confidence, a feeling of triumph and a better understanding of the concept of integration. They may need to explore a little when

they graph the function.

Example 3. Let

$$f := x \to \left(1 + \frac{1}{x}\right)^x$$

a) Graph the function

b)Estimate

$$\lim_{x\to\infty}\left(1 + \frac{1}{x}\right)^x$$

c) Complete the following table

x | 1 2 5 10 100 1000 10000

f |

Again, through the machine computation, students didn't have trouble to find answers for a) and b), but when they started to calculate f(1000), the computer gave them a outrageous number.

What was going on? Looking back to part b) and using the concept of the limit, the students knew the answer couldn't be right. They had to try different approaches, like writing $1/x$ as x^{-1}, or put a paranthesis around $1/x$, ... Finally, they decided to put 1/1000 as 0.001, and this time they were right!

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> (1+0.001)^1000;
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$$2.716923932$$

These kind of exercises not only provided opportunity to reinforce the concept of limits, but also enhanced students' arithmetic and algebra skills.

Example 4. Choose two positive numbers for a and b.
Use these two values to define a function.
a) Write down two equations to be solved in order to find the critical points of f.
b) To solve for the critical points, the above two equations can be combined to a single equation involving y only.
Write down such an equation.
c) Use Maple to solve for y.
d) Determine the nature of the critical points.
e) Determine the values of x and y, with which to see how f behaves near the first critical points from the picture of f.
f) Define a new function g as f+3. By the previous calculation, what are the critical points of g and their nature?
Explain.

This example offers a good chance for students to look at the whole picture of finding the extrema for a given multi-variables function using the Second Derivatives Test.Typically, students would have to struggle in the computation swamp and try to avoid careless mistakes. Some times they would simply get lost in the middle of the process. Now Maple can free them from the tedious calculation so that they can keep a clear mind on the task and can even make a connection back to the derivative rules.

The following two examples will help students to visualize solids of revolution.

Example 5. Find the volume of the solid by rotating about the x-axis the region under the curve

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> f:= (x) -> (5*x)^(1/2);
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$$f := x \to \sqrt{5}\,\sqrt{x}$$

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>
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from x=0 to x=1.

Example 6. Find the volume obtained by revolving the region bouned by

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> eq1 :=y=x^2/3;
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$$eq1 := y = \frac{1}{3}x^2$$

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> eq2 :=y=2*x;
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$$eq2 := y = 2x$$

about the line y = 15 in the x-y plane.

3. Conclusion

In looking at the substantial changes that technology has brought in the recent years, we believe that instruction in mathematics will have to catch up with the new circumstances or else become increasingly irrelevant. With added pressure from the rapid development of information super highway, it is even more demanding to train our students to think clearly, critically, constructively, and creatively about the problems they might encounter in the real world. It is our job to help students to gain the ability that will enable them to use mathematical methods and tools whenever they seem appropriate and helpful. To this end, computer-oriented mathematics courses, focusing on coorperative learning, problems solving, and investigative learning and writing are an important part of the education for our students.

On the other hand, through our experience of integrating Maple into teaching the calculus sequence, we feel questions like these still need to be answered. a) How much technology is too much? We don't want students become too dependent on it. b) If something went wrong during the process of using Maple, students may panic because they don't know whether it was a math problem or they did something wrong. How do we help students to identify such problems and solve them efficiently? We hope we will be able to find satisfing answers in the near future.

4. References

1. William E. Boyce, New Directions in Elementary Differential Equations, College Mathematics Journal 25:5 (1994) 364 - 371.

2. John S. Devitt, Calculus with Maple V, Brooks/Cole Publishing Company, 1993.

3. C.K. Cheung and John Harer, Multivariable Calculus with Maple V, Preliminary Edition, John Wiley and Sons,Inc, 1994.

4. Michael C. Reed, SIAM News 27:7 (1994).

5. Joe Brown, Report to the Calculus Consortium, The Appalachian Journal of School and College Mathematics, Fall 1993 Vol.1, Issue 1.