Reducing Tedium in Teaching and Learning

ICTCM 95

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Overview and Rationale

At Southwestern University, a liberal arts institution for some 1200 students, we have been cautious in embracing technology in teaching and learning. I have been concerned that students might let the computer do all the thinking for them. I have learned how to use the high powered computer algebra system Mathematica in such a way that students still make the decisions for procedures for themselves but allow the computer to take care of the detail.

I have seen these techniques alleviate some of the common learning difficulties of students. For instance, many students become overwhelmed by details involved in such procedures as partial fractions decomposition. Because of this, they have other problems, too. They are frustrated by sensitivity to error as seen in Guass elimination by hand. Some students fail to see the main ideas of a topic or are unable to perform what are essentially algorithms, such as in finding eigensystems. For others, math seems "magic" and arbitrary when done by others, as in finding series solutions to differential equations.

By adopting the "top-down" approach in leaving the details to the computer, students can avoid the technology crutch and approach the goal of developing determination and mathematical maturity to perform mathematics without the technology. It allows me to specify the level of dependence upon the computer.

The examples I present can be used as computer laboratories for students, thereby aiding in their learning. I have used these techniques also to present material to students in an organized fashion. I can generate many examples quickly and cleanly. I can use technology to trace student errors; for instance, on an exam, a student makes a mistake in a procedure. I can give partial credit for that portion and quickly check if the remainder of the problem would be correct. I have also used this in the office when a student comes in for help. I have told students they may check their own work using technology rather than asking someone else to help trace an error. I can quickly generate keys to problem sets and exams as well. As an added bonus, students don't have to read my handwriting, and I don't have to decipher theirs. I am still trying to find the best balance between technology and traditional methods, but these are some of the ways I have found of using technology to reduce tedium in teaching as well as in learning of mathematics.

The following are modules which illustrate the differences in full automation and in exercising more control over the computer algebra system. Each illustrates at least the point listed below.

Partial Fractions - variety of techniques for solving equations
Polynomial Division - algorithmic approach is apparent
Eigensystems - includes manual Guass elimination
Series Solution - 10 terms no more difficult than 3

Partial Fractions

Mathematica commands used:
Apart Together
Numerator
Collect Select FreeQ
&
Coefficient
Solve Integrate

Automatic

Let's start with a rational expression in reduced form.

```
ratl = (x^2 + 3x - 5) / ( (x + 1)^2 (x^2 + 9) )
                  2
  -5 + 3 x + x
------------------------------
        2        2
(1 + x)  (9 + x )
```

Mathematica can perform a partial fractions decomposition.

```
Apart[ratl]
    -7              1            83 + 2 x
---------------- - --------------- + -----------------
          2       25 (1 + x)                2
10 (1 + x)                          50 (9 + x )
```

Manual

Initial work.

Assume the proper pfd form

```
pfdForm = a1/(x + 1) + a2/(x + 1)^2 + (b x + c)/(x^2 + 9)
```

$$\frac{a2}{(1+x)^2} + \frac{a1}{1+x} + \frac{c + b\,x}{9+x^2}$$

Get a common denominator for the form

```
Together[%]
```

$$(9\,a1 + 9\,a2 + c + 9\,a1\,x + b\,x + 2\,c\,x + a1\,x^2 + a2\,x^2 + 2\,b\,x^2 + c\,x^2 + a1\,x^3 + b\,x^3) \,/\, ((1+x)^2\,(9+x^2))$$

Peel off the numerator.

```
Numerator[%]
```

$$9\,a1 + 9\,a2 + c + 9\,a1\,x + b\,x + 2\,c\,x + a1\,x^2 + a2\,x^2 + 2\,b\,x^2 + c\,x^2 + a1\,x^3 + b\,x^3$$

Collect like terms.

```
pfdNumerator = Collect[%,x]
```

$$9\,a1 + 9\,a2 + c + (9\,a1 + b + 2\,c)\,x + (a1 + a2 + 2\,b + c)\,x^2 + (a1 + b)\,x^3$$

Manual set-up of equations

The constant terms are free of any x's.

```
c0 = Select[pfdNumerator, FreeQ[#,x]&]
```

9 a1 + 9 a2 + c

Coefficients of higher powers of x may be peeled off, too.

```
c1 = Coefficient[pfdNumerator, x]
```

9 a1 + b + 2 c

```
c2 = Coefficient[pfdNumerator, x^2]
```

a1 + a2 + 2 b + c

```
c3 = Coefficient[pfdNumerator, x^3]
```

a1 + b

Mannually setting up the equations and solving them automatically.

```
eqn0 = (c0 == -5);      eqn1 = (c1 == 3);
eqn2 = (c2 == 1);       eqn3 = (c3 == 0);
```

■ Automatic solving; uses.

```
Solve[{eqn0,eqn1,eqn2,eqn3},{a1,a2,b,c}]
```

$$\left\{\left\{a2 \to -\left(\frac{7}{10}\right),\ b \to \frac{1}{25},\ c \to \frac{83}{50},\ a1 \to -\left(\frac{1}{25}\right)\right\}\right\}$$

```
pfdForm/.{a2 -> -7/10, b -> 1/25, c -> 83/50, a1 -> -1/25}
```

$$\frac{-7}{10\,(1+x)^2} - \frac{1}{25\,(1+x)} + \frac{\frac{83}{50} + \frac{x}{25}}{9 + x^2}$$

Such a decomposition is needed in Calculus and Differential Equations.

```
Integrate[%,x]
```

$$\frac{7}{10\,(1+x)} - \frac{83\,\mathrm{ArcTan}[\frac{3}{x}]}{150} - \frac{\mathrm{Log}[1+x]}{25} + \frac{\mathrm{Log}[9+x^2]}{50}$$

■ Equations by evaluations

```
num = Numerator[ratl]
                2
-5 + 3 x + x
pfdNumerator/.x->-1
10 a2
num/.x->-1
-7
eq1 = % == %%
-7 == 10 a2
pfdNumerator/.x->0
9 a1 + 9 a2 + c
num/.x->0
-5
eq2 = % == %%
-5 == 9 a1 + 9 a2 + c
pfdNumerator/.x->1
20 a1 + 10 a2 + 4 b + 4 c
num/.x->1
-1
eq3 = % == %%
-1 == 20 a1 + 10 a2 + 4 b + 4 c
pfdNumerator/.x->2
9 a1 + 9 a2 + 8 (a1 + b) + c + 4 (a1 + a2 + 2 b + c) +

  2 (9 a1 + b + 2 c)
num/.x->2
5
eq4 = % == %%
5 == 9 a1 + 9 a2 + 8 (a1 + b) + c + 4 (a1 + a2 + 2 b + c) +

   2 (9 a1 + b + 2 c)
Solve[{eq1,eq2,eq3,eq4},{a1,a2,b,c}]
       1            1          83          7
{{b -> --, a1 -> -(--), c -> --, a2 -> -(--)}}
       25           25         50          10
```

Division of Polynomials

Mathematica commands used:
Factor Numerator
Denominator
Exponent
Coefficient Expand

The Set-up

Define the original numerator and denominator, free of any common factors.

```
num=2x^4+1
```

$1 + 2x^4$

```
den=x^2+1
```

$1 + x^2$

Of course, you can also eliminate common factors first. Suppose you begin with

```
ratl = (-40 + 8*x - 5*x^2 + x^3 - 80*x^4 + 16*x^5 - 10*x^6 + 2*x^7)/
    (-40 + 8*x - 45*x^2 + 9*x^3 - 5*x^4 + x^5)
```

$$\frac{-40 + 8x - 5x^2 + x^3 - 80x^4 + 16x^5 - 10x^6 + 2x^7}{-40 + 8x - 45x^2 + 9x^3 - 5x^4 + x^5}$$

```
ratl2 = Factor[ratl]
```

$$\frac{1 + 2x^4}{1 + x^2}$$

Then you can get the numerator and denominator:

```
num = Numerator[ratl2]
den = Denominator[ratl2]
```

$1 + 2x^4$

$1 + x^2$

For the first iteration of the algorithm, initialize the whole part to be zero.
Find the order and corresponding coefficients of the denominator and numerator.

```
whole=0
```

0

```
pNum=Exponent[num,x]
```

4

```
pDen=Exponent[den,x]
```

2

Since the numerator has a higher order than the denominator, we need to divide.

```
CoeDen=Coefficient[den,x^pDen]
```

1

```
CoeNum=Coefficient[num,x^pNum]
```

2

Compute the current factor of the whole portion, and update the whole portion.

```
fac=Expand[CoeNum/CoeDen x^(pNum-pDen)]
```

$2 x^2$

```
whole=whole+fac
```

$2 x^2$

Compute the reduced numerataor

```
num = Expand[num-fac den]
```

$1 - 2 x^2$

Second Iteration

Find the order and corresponding coefficient of the new numerator.

```
pNum=Exponent[num,x]
```

2

The new portion is still improper.

```
CoeNum=Coefficient[num,x^pNum]
```

-2

Compute the current factor and add it to the whole portion of the reduction.

```
fac=Expand[CoeNum/CoeDen x^(pNum-pDen)]
```

-2

```
whole=whole+fac
```

$-2 + 2 x^2$

```
num=Expand[num-fac den]
```

3

Now we have the reduced form.

```
whole + num/den
```

$$-2 + 2 x^2 + \frac{3}{1 + x^2}$$

Eigensystems

Mathematica commands used:
Eigenvalues
Eigenvectors
Eigensystem
IdentityMatrix
MatrixForm
Det
Solve
Nullspace

Consider the following matrix. (Boyce&DriPrima 7.5 #17 3rd ed.)

```
A = {{3,2,2},{1,4,1},{-2,-4,-1}}
{{3, 2, 2}, {1, 4, 1}, {-2, -4, -1}}
```

Automatic computation

You can have Mathematica automatically compute the eigensystem like this:

```
Eigenvalues[A]
{1, 2, 3}
Eigenvectors[A]
{{-1, 0, 1}, {-2, 1, 0}, {0, -1, 1}}
```

or like this

```
Eigensystem[A]
{{1, 2, 3}, {{-1, 0, 1}, {-2, 1, 0}, {0, -1, 1}}}
```

Semi-manual calculations

Alternatively, you can do it somewhat manually. Defining a function is somewhat simpler but unnecessary.

```
f[t_] = A - t IdentityMatrix[3]
{{3 - t, 2, 2}, {1, 4 - t, 1}, {-2, -4, -1 - t}}
```

You can view any matrix more naturally.

```
MatrixForm[%]
3 - t    2        2

1        4 - t    1

-2       -4       -1 - t
```

eigenvalues

```
d = Det[%]
```

$6 - 11 t + 6 t^2 - t^3$

```
Solve[d==0,t]
{{t -> 1}, {t -> 2}, {t -> 3}}
```

eigenvector as solution to vector equation

```
A2 = f[1]
{{2, 2, 2}, {1, 3, 1}, {-2, -4, -2}}
A2 .{x2,y2,z2}
{2 x2 + 2 y2 + 2 z2, x2 + 3 y2 + z2, -2 x2 - 4 y2 - 2 z2}
Solve[A2 .{x2,y2,z2} == {0,0,0},{x2,y2,z2}]
{{x2 -> -z2, y2 -> 0}}
```

Note that Mathematica will allow you to just specify 0 rather than a vector, but I think students need to specify it precisely.

```
Solve[A2 .{x2,y2,z2} == 0,{x2,y2,z2}]
{{x2 -> -z2, y2 -> 0}}
eigenvector is {-z2,0,z2} or z2{-1,0,1}  .
```

eigenvector via manual Guass elimination

```
mat = f[3]
{{0, 2, 2}, {1, 1, 1}, {-2, -4, -4}}
mat2 = {mat[[2]],  mat[[1]],   mat[[3]] + 2 mat[[2]]}
{{1, 1, 1}, {0, 2, 2}, {0, -2, -2}}
mat3 = {mat2[[1]] - 1/2 mat2[[2]], 1/2 mat2[[2]],
        mat2[[2]] + mat2[[3]]}
{{1, 0, 0}, {0, 1, 1}, {0, 0, 0}}
```

Thus, for the eigenvector {x3, y3, z3}, we must have x3 = 0 and y3 = - z3.
That is, {0, -z3, z3} or {0, -1, 1}.

■ eigenvector as basis for the nullspace

```
A3 = f[2]
{{1, 2, 2}, {1, 2, 1}, {-2, -4, -3}}
NullSpace[A3]
{{-2, 1, 0}}
```

Series Solutions

Mathematica commans used:
Sum
O
D
LogicalExpand
Solve
Collect
Table
Coefficient
Denominator
Factor Integer
Factorial

Series solutions near an ordinary point for a second order, linear, homogenous DE
y'' + y = 0

3 terms in each of 2 linearly independent solns

First state the form of the solution to a desired number of terms.

y=Sum[a[i] x^i, {i,0,6}] + O[x]^7

$$a[0] + a[1]\, x + a[2]\, x^2 + a[3]\, x^3 + a[4]\, x^4 + a[5]\, x^5 + a[6]\, x^6 + O[x]^7$$

Calculate the necessary derivatives

y1=D[y,x]

$$a[1] + 2\, a[2]\, x + 3\, a[3]\, x^2 + 4\, a[4]\, x^3 + 5\, a[5]\, x^4 + 6\, a[6]\, x^5 + O[x]^6$$

y2=D[y1,x]

$$2\, a[2] + 6\, a[3]\, x + 12\, a[4]\, x^2 + 20\, a[5]\, x^3 + 30\, a[6]\, x^4 + O[x]^5$$

Substitute y and its derivatives into the differential equation.

lhs=y2+y

$$(a[0] + 2\, a[2]) + (a[1] + 6\, a[3])\, x + (a[2] + 12\, a[4])\, x^2 + (a[3] + 20\, a[5])\, x^3 + (a[4] + 30\, a[6])\, x^4 + O[x]^5$$

Equate the coefficients for corresponding powers of x.

LogicalExpand[lhs==0]

```
a[0] + 2 a[2] == 0 && a[1] + 6 a[3] == 0 &&

  a[2] + 12 a[4] == 0 && a[3] + 20 a[5] == 0 &&

  a[4] + 30 a[6] == 0
```

Since this is second order, we should be able to solve for all but the first two terms in the series solution.

Solve[%,{a[2],a[3],a[4],a[5],a[6]}]

$$\{\{a[5] \to \frac{a[1]}{120},\ a[6] \to \frac{-a[0]}{720},\ a[2] \to \frac{-a[0]}{2},\ a[3] \to \frac{-a[1]}{6},\ a[4] \to \frac{a[0]}{24}\}\}$$

Substitute these back into y.

```
{ysoln}=y/.%
```

$$\{a[0] + a[1]\, x - \frac{a[0]\, x^2}{2} - \frac{a[1]\, x^3}{6} + \frac{a[0]\, x^4}{24} + \frac{a[1]\, x^5}{120} - \frac{a[0]\, x^6}{720} + O[x]^7\}$$

Collect the terms for the two linearly independent series solutions.

```
ysoln=Collect[ysoln,{a[0],a[1]}]
```

$$\left(1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720}\right) a[0] + \left(x - \frac{x^3}{6} + \frac{x^5}{120}\right) a[1]$$

10 terms in each solution

Of course, this same technique is used if we wish to take these to 10 terms each.

```
y=Sum[a[i] x^i, {i,0,20}] + O[x]^21
```

$$a[0] + a[1]\, x + a[2]\, x^{2} + a[3]\, x^{3} + a[4]\, x^{4} + a[5]\, x^{5} + a[6]\, x^{6} + a[7]\, x^{7} + a[8]\, x^{8} + a[9]\, x^{9} + a[10]\, x^{10} + a[11]\, x^{11} + a[12]\, x^{12} + a[13]\, x^{13} + a[14]\, x^{14} + a[15]\, x^{15} + a[16]\, x^{16} + a[17]\, x^{17} + a[18]\, x^{18} + a[19]\, x^{19} + a[20]\, x^{20} + O[x]^{21}$$

```
y1=D[y,x]
```

$$a[1] + 2\, a[2]\, x + 3\, a[3]\, x^{2} + 4\, a[4]\, x^{3} + 5\, a[5]\, x^{4} + 6\, a[6]\, x^{5} + 7\, a[7]\, x^{6} + 8\, a[8]\, x^{7} + 9\, a[9]\, x^{8} + 10\, a[10]\, x^{9} + 11\, a[11]\, x^{10} + 12\, a[12]\, x^{11} + 13\, a[13]\, x^{12} + 14\, a[14]\, x^{13} + 15\, a[15]\, x^{14} + 16\, a[16]\, x^{15} + 17\, a[17]\, x^{16} + 18\, a[18]\, x^{17} + 19\, a[19]\, x^{18} + 20\, a[20]\, x^{19} + O[x]^{20}$$

```
y2=D[y1,x]
```

$$2\, a[2] + 6\, a[3]\, x + 12\, a[4]\, x^{2} + 20\, a[5]\, x^{3} + 30\, a[6]\, x^{4} + 42\, a[7]\, x^{5} + 56\, a[8]\, x^{6} + 72\, a[9]\, x^{7} + 90\, a[10]\, x^{8} + 110\, a[11]\, x^{9} + 132\, a[12]\, x^{10} + 156\, a[13]\, x^{11} + 182\, a[14]\, x^{12} + 210\, a[15]\, x^{13} + 240\, a[16]\, x^{14} + 272\, a[17]\, x^{15} + 306\, a[18]\, x^{16} + 342\, a[19]\, x^{17} + 380\, a[20]\, x^{18} + O[x]^{19}$$

```
lhs=y2+y
```

```
(a[0] + 2 a[2]) + (a[1] + 6 a[3]) x + (a[2] + 12 a[4]) x^2 +
  (a[3] + 20 a[5]) x^3 + (a[4] + 30 a[6]) x^4 +
  (a[5] + 42 a[7]) x^5 + (a[6] + 56 a[8]) x^6 +
  (a[7] + 72 a[9]) x^7 + (a[8] + 90 a[10]) x^8 +
  (a[9] + 110 a[11]) x^9 + (a[10] + 132 a[12]) x^10 +
  (a[11] + 156 a[13]) x^11 + (a[12] + 182 a[14]) x^12 +
  (a[13] + 210 a[15]) x^13 + (a[14] + 240 a[16]) x^14 +
  (a[15] + 272 a[17]) x^15 + (a[16] + 306 a[18]) x^16 +
  (a[17] + 342 a[19]) x^17 + (a[18] + 380 a[20]) x^18 + O[x]^19
```

```
eqnset=LogicalExpand[lhs==0]
```

```
a[0] + 2 a[2] == 0 && a[1] + 6 a[3] == 0 &&
  a[2] + 12 a[4] == 0 && a[3] + 20 a[5] == 0 &&
  a[4] + 30 a[6] == 0 && a[5] + 42 a[7] == 0 &&
  a[6] + 56 a[8] == 0 && a[7] + 72 a[9] == 0 &&
  a[8] + 90 a[10] == 0 && a[9] + 110 a[11] == 0 &&
  a[10] + 132 a[12] == 0 && a[11] + 156 a[13] == 0 &&
  a[12] + 182 a[14] == 0 && a[13] + 210 a[15] == 0 &&
  a[14] + 240 a[16] == 0 && a[15] + 272 a[17] == 0 &&
  a[16] + 306 a[18] == 0 && a[17] + 342 a[19] == 0 &&
  a[18] + 380 a[20] == 0
```

```
varset=Table[a[i],{i,2,20}]
```

```
{a[2], a[3], a[4], a[5], a[6], a[7], a[8], a[9], a[10], a[11],
  a[12], a[13], a[14], a[15], a[16], a[17], a[18], a[19], a[20]}
```

Solve[eqnset,varset]

$$\{\{a[19] \to \frac{-a[1]}{121645100408832000},\ a[20] \to \frac{a[0]}{2432902008176640000},$$
$$a[2] \to \frac{-a[0]}{2},\ a[3] \to \frac{-a[1]}{6},\ a[4] \to \frac{a[0]}{24},\ a[5] \to \frac{a[1]}{120},$$
$$a[6] \to \frac{-a[0]}{720},\ a[7] \to \frac{-a[1]}{5040},\ a[8] \to \frac{a[0]}{40320},\ a[9] \to \frac{a[1]}{362880},$$
$$a[10] \to \frac{-a[0]}{3628800},\ a[11] \to \frac{-a[1]}{39916800},\ a[12] \to \frac{a[0]}{479001600},$$
$$a[13] \to \frac{a[1]}{6227020800},\ a[14] \to \frac{-a[0]}{87178291200},$$
$$a[15] \to \frac{-a[1]}{1307674368000},\ a[16] \to \frac{a[0]}{20922789888000},$$
$$a[17] \to \frac{a[1]}{355687428096000},\ a[18] \to \frac{-a[0]}{6402373705728000}\}\}$$

{ysoln}=y/.%

$$\{a[0] + a[1]\,x - \frac{a[0]\,x^2}{2} - \frac{a[1]\,x^3}{6} + \frac{a[0]\,x^4}{24} + \frac{a[1]\,x^5}{120} -$$
$$\frac{a[0]\,x^6}{720} - \frac{a[1]\,x^7}{5040} + \frac{a[0]\,x^8}{40320} + \frac{a[1]\,x^9}{362880} - \frac{a[0]\,x^{10}}{3628800} - \frac{a[1]\,x^{11}}{39916800} +$$
$$\frac{a[0]\,x^{12}}{479001600} + \frac{a[1]\,x^{13}}{6227020800} - \frac{a[0]\,x^{14}}{87178291200} - \frac{a[1]\,x^{15}}{1307674368000} +$$
$$\frac{a[0]\,x^{16}}{20922789888000} + \frac{a[1]\,x^{17}}{355687428096000} - \frac{a[0]\,x^{18}}{6402373705728000} -$$
$$\frac{a[1]\,x^{19}}{121645100408832000} + \frac{a[0]\,x^{20}}{2432902008176640000} + O[x]^{21}\}$$

ysoln=Collect[ysoln,{a[0],a[1]}]

$$\left(1-\frac{x^{2}}{2}+\frac{x^{4}}{24}-\frac{x^{6}}{720}+\frac{x^{8}}{40320}-\frac{x^{10}}{3628800}+\frac{x^{12}}{479001600}-\frac{x^{14}}{87178291200}+\frac{x^{16}}{20922789888000}-\frac{x^{18}}{6402373705728000}+\frac{x^{20}}{2432902008176640000}\right) a[0] + \left(x-\frac{x^{3}}{6}+\frac{x^{5}}{120}-\frac{x^{7}}{5040}+\frac{x^{9}}{362880}-\frac{x^{11}}{39916800}+\frac{x^{13}}{6227020800}-\frac{x^{15}}{1307674368000}+\frac{x^{17}}{355687428096000}-\frac{x^{19}}{121645100408832000}\right) a[1]$$

u1=Coefficient[ysoln,a[0]]

$$1-\frac{x^{2}}{2}+\frac{x^{4}}{24}-\frac{x^{6}}{720}+\frac{x^{8}}{40320}-\frac{x^{10}}{3628800}+\frac{x^{12}}{479001600}-\frac{x^{14}}{87178291200}+\frac{x^{16}}{20922789888000}-\frac{x^{18}}{6402373705728000}+\frac{x^{20}}{2432902008176640000}$$

u2=Coefficient[ysoln,a[1]]

$$x-\frac{x^{3}}{6}+\frac{x^{5}}{120}-\frac{x^{7}}{5040}+\frac{x^{9}}{362880}-\frac{x^{11}}{39916800}+\frac{x^{13}}{6227020800}-\frac{x^{15}}{1307674368000}+\frac{x^{17}}{355687428096000}-\frac{x^{19}}{121645100408832000}$$

Looking for patterns in the coefficients

Let's look at u1. An obvious pattern is (-1) * something x^(2n). Maybe factoring the coefficients will show something.

```
t=Table[Coefficient[u1,x^(2n)],{n,0,10}]
```

$$\left\{1, -\left(\frac{1}{2}\right), \frac{1}{24}, -\left(\frac{1}{720}\right), \frac{1}{40320}, -\left(\frac{1}{3628800}\right), \frac{1}{479001600}, -\left(\frac{1}{87178291200}\right), \frac{1}{20922789888000}, -\left(\frac{1}{6402373705728000}\right), \frac{1}{2432902008176640000}\right\}$$

```
dlist=Denominator[t]
```

```
{1, 2, 24, 720, 40320, 3628800, 479001600, 87178291200,

  20922789888000, 6402373705728000, 2432902008176640000}
```

One thing you can try is getting prime factors. In the following, we see that 24 = 2^3 * 3; 720=2^4 * 3^2 * 5, etc.

```
flist=FactorInteger[dlist]
```

```
{{}, {{2, 1}}, {{2, 3}, {3, 1}}, {{2, 4}, {3, 2}, {5, 1}},

  {{2, 7}, {3, 2}, {5, 1}, {7, 1}},

  {{2, 8}, {3, 4}, {5, 2}, {7, 1}},

  {{2, 10}, {3, 5}, {5, 2}, {7, 1}, {11, 1}},

  {{2, 11}, {3, 5}, {5, 2}, {7, 2}, {11, 1}, {13, 1}},

  {{2, 15}, {3, 6}, {5, 3}, {7, 2}, {11, 1}, {13, 1}},

  {{2, 16}, {3, 8}, {5, 3}, {7, 2}, {11, 1}, {13, 1}, {17, 1}},

  {{2, 18}, {3, 8}, {5, 4}, {7, 2}, {11, 1}, {13, 1}, {17, 1},

   {19, 1}}}
```

You might try successive quotients.

```
flist2=Table[dlist[[i+1]]/dlist[[i]],{i,1,10}]
```

```
{2, 12, 30, 56, 90, 132, 182, 240, 306, 380}
```

```
FactorInteger[%]
```

{{{2, 1}}, {{2, 2}, {3, 1}}, {{2, 1}, {3, 1}, {5, 1}},
{{2, 3}, {7, 1}}, {{2, 1}, {3, 2}, {5, 1}},
{{2, 2}, {3, 1}, {11, 1}}, {{2, 1}, {7, 1}, {13, 1}},
{{2, 4}, {3, 1}, {5, 1}}, {{2, 1}, {3, 2}, {17, 1}},
{{2, 2}, {5, 1}, {19, 1}}}

And you can always look for factorials.

```
Table[Factorial[n],{n,2,20}]
```

{2, 6, 24, 120, 720, 5040, 40320, 362880, 3628800, 39916800,
479001600, 6227020800, 87178291200, 1307674368000,
20922789888000, 355687428096000, 6402373705728000,
121645100408832000, 2432902008176640000}

From this one can see that the even powers of x get even factorials and the odd powers get odd factorials. This can be double checked.

```
v1=Sum[(-1)^n/Factorial[2n] x^(2n),{n,0,10}]
```

$$1 - \frac{x^2}{2} + \frac{x^4}{24} - \frac{x^6}{720} + \frac{x^8}{40320} - \frac{x^{10}}{3628800} + \frac{x^{12}}{479001600} - \frac{x^{14}}{87178291200} + \frac{x^{16}}{20922789888000} - \frac{x^{18}}{6402373705728000} + \frac{x^{20}}{2432902008176640000}$$

```
v2=Sum[(-1)^n/Factorial[2n+1] x^(2n+1),{n,0,10}]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + \frac{x^9}{362880} - \frac{x^{11}}{39916800} + \frac{x^{13}}{6227020800} - \frac{x^{15}}{1307674368000} + \frac{x^{17}}{355687428096000} - \frac{x^{19}}{121645100408832000} + \frac{x^{21}}{51090942171709440000}$$

```
u1-v1
```

0

u2-v2

```
         21
       -x
______________________
5109094217170944000
```