Linear Algebra and the TI-85

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In this presentation we will discuss Leontief's economic model and the accessibility index of a network. Most texts give application examples and exercises involving 2 x 2 or 3 x 3 matrices only. Now with the help of the TI-85 and other math software we can consider examples involving higher dimensional matrices.

Leontief's Economic Model:

Wassily Leontief developed the famous input-output economic model in 1949 for which he was awarded the Nobel prize in 1973. This model described the interrelations betweeen various sectors in an economy and involved a system of 500 linear equations in 500 unknowns. In [2], he applied this model to 170 sectors of world economy. This model has become a standard tool for investigating economic structures of cities, states and countries. In 1982, North Dakota Department of Agriculutre developed an input-output model involving 17 sectors of its economy. The North Dakota model was successfully adapted by Minnesota, Montana and Wyoming in 1983. The Minnesota economy was divided into 20 sectors and so gave rise to a 20 x 20 matrix [1].

The input-output model uses the following equation:

Total output = intersector portion of the output
+ demand of the open sector.

This gives rise to a matrix equation of the form:

$$\bar{X} = A\bar{X} + D$$

Where x_i = total output of ith sector.
a_{ij} = amount of output of the ith sector needed by the jth sector to produce one dollar's worth output.
d_i = demand of the open sector from the ith sector.

In a closed model, D = 0 and $\sum_{i=1}^{n} a_{ij} = 1$. In both cases, Frobenius-Perron theorem gives a unique nonnegative solution x.

We consider the following input-output matrix A:

$$
\begin{array}{ll}
1. & \text{Auto} \\
2. & \text{Steel} \\
3. & \text{Electricity} \\
4. & \text{Coal} \\
5. & \text{Chemical}
\end{array}
\qquad
\begin{array}{ccccc}
1 & 2 & 3 & 4 & 5
\end{array}
$$
$$
\begin{bmatrix}
0.15 & 0.10 & 0.05 & 0.05 & 0.10 \\
0.40 & 0.20 & 0.10 & 0.10 & 0.10 \\
0.10 & 0.25 & 0.20 & 0.10 & 0.20 \\
0.10 & 0.20 & 0.30 & 0.15 & 0.10 \\
0.05 & 0.10 & 0.05 & 0.02 & 0.05
\end{bmatrix}
$$

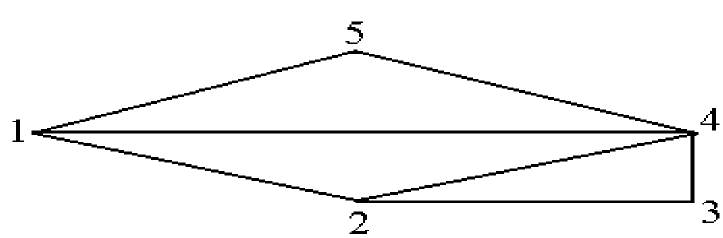
and demand matrix $D = \begin{bmatrix} 2 \\ 1 \\ 0 \\ .5 \\ .2 \end{bmatrix}$ billion dollars.

Then the total output is $(I - A)^{-1}D = \begin{bmatrix} 3.5 \\ 3.52 \\ 2.05 \\ 2.62 \\ 0.91 \end{bmatrix}$ billion dollars.

Accessibility Index of a Network:

The accessibility index of a network is of primary interest in many physical applications of geography and history since 1960. In geography, one considers a transportation network involving railway lines, highways, airline routes or canals. This network can be represented by a graph. The dominance of Moscow has been attributed by Russian historians to its strategic position on trade routes. Recent techniques of graphy theory have been used by Pitt, in [4], to challenge this theory. Moscow ranks sixth when these techniques are applied to twelfth century trade routes of Russia. This study gives rise to a 39 x 39 matrix. For details see [4] and [5].

We consider the following network of 2-way road:



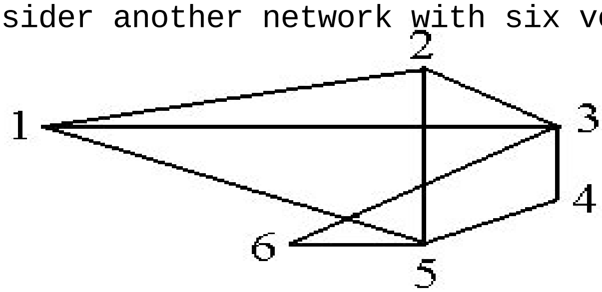
Its adjancy matrix A and augmented adjancy matrix B = A + I are given by:

$$A = \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Using a TI-85 program, see [3], we find the dominant eigenvalue • = 3.94 and a corresponding eigenvector v = (1.85, 1.85, 1.38, 2.20, 1.38) of the matrix B. The fourth component of v is the largest and so the vertex 4 has the highest accessibility. The geographers divide v by the sum of its components to define the Gould index of a vertex. Thus

Vertex	Gould index
1	.21
2	.21
3	.16
4	.25
5	.16

We consider another network with six vertices.



Its dominant eigenvalue is • = 4.14 and a corresponding eigenvector is v = (1.81, 1.81, 1.94, 1.24, 1.94, 1.24). For more applications of graphy theory see [7].

References

1. Randal C. Coon et al; Expansion and Adaptation of the North Dakota Economic - Demographic Assessment Model for Minnesota. North Dakota State University, 1984.
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4. F. R. Pitts, A Graph Theoretical Approach to Historical Geography. Professional Geography, 1965.
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6. Gareth Williams; Linear Algebra with Applications. William C. Brown Publishers, 1991.
7. R. J. Wilson and L. W. Beineke; Applications of Graph Theory. Academic Press, 1979.