INTERACTIVE LABORATORIES FOR TEACHING APPLIED MATHEMATICS

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1. Introduction to Interactive Laboratories

Applied Mathematics, a course designed for students majoring in sciences and engineering, covers integral transforms, Fourier analysis, and partial differential equations. Many notions in this course are difficult for students to comprehend. Over the years, this course was taught in a traditional chalk-and-board way, and some frustrating shortcomings have emerged in the students' learning.

- Chalk and board lack the means to present concepts in a vivid and graphic way.
- Lengthy computations in classroom deviate students from learning the essence of the materials.
- Passive learning dampens the students' enthusiasm for creative thinking and realistic application.

The rapid development of computer technology has made it possible to teach mathematics by developing *interactive mathematics laboratory*. A mathematics laboratory is equipped with new generation of computers and other accessories. The "experiments" in the laboratory is to explore *interactive electronic texts*, that can be described as computer documents from which symbolic, numeric and graphical tools can be invoked together with features that support experimenting with as many examples as one wants to understand and explore ideas.

In 1992 an Interactive Mathematics Text Project (IMTP) laboratory classroom is opened in our department thanks to a grant from NSF, MAA and IBM. This classroom is equipped with sixteen IBM 486 PC's with various software that includes *Mathematica* and *Maple*. The resources in the IMTP laboratory have allowed us to experiment innovatively with new approaches to the undergraduate mathematics curriculums ranging from college algebra to differential equations.

In order to develop interactive laboratories for *Applied Mathematics*, a collection of "high level functions" (smart functions or interactive functions) was created using *Maple* software package. These functions enable students to utilize the excellent symbolic and graphic capabilities of *Maple* without being entangled in the syntax maze of the language.

The selection of *Maple* is based on three factors: its enormous symbolic, computing and graphic power; the author's success in developing interactive texts for calculus in *Maple*; and the availability of *Maple* in our IMTP laboratory.

In Section 2, we shall have a general look on the interactive laboratories of *Applied Mathematics*.

2. Applied Mathematics Laboratories

The main purpose of the project of *Applied Mathematics Laboratories* is to develop and implement a curriculum that promotes a laboratory-based, collaborative, and highly interactive model of instruction, and encourages active student participation in the learning process. The course is to be taught in a computer laboratory setting with the emphasis of the use of technology for visualization, numerical computation, and symbolic manipulation.

In the *Applied Mathematics* laboratories, students have the unique opportunity to explore, revise, and finally understand the concepts by taking a series of carefully designed activities, which include answering questions, graphically explaining the concepts or solving a problem related the concepts they just learned. Finally the students will be given a set of exercises or projects of application problems.

The following is a list of main features of the *Applied Mathematics* laboratories.

- Concepts and examples are presented graphically (including animation), analytically and by combination of both. In the laboratory of Fourier series, students not only can obtain the Fourier expansion of a function $f(x)$ analytically, but also can see a vivid animation of how partial sums S_n of the corresponding Fourier series approach to $f(x)$ as n increases.

- Realistic applications are used from a variety of areas to introduce and motivate mathematical concepts. For example, the animation of the vibration of a violin string (one dimension) or a stretched membrane, such as a drumhead (two dimensions) will be presented to derive the wave equations.

- Syntax jargons of computer language are avoided so that even students with minimum computer background can navigate through the laboratory. Most questions posted on the screen are multiple choices. Therefore, students only need to select the correct answer by inputting a number or a letter.

- User-friendly designs are emphasized. Often students have difficulties to solve a problem, or can not remember some formulas or concepts needed to answer a question. In that situation hints will be given, or previously learned knowledges will be reviewed on the screen at the students' request.

- Students' inputs, especially their answers are checked immediately, and a message will show on the screen, telling them whether their input is correct or wrong.

Currently, about 60% of the material in *Applied Mathematics*, including *Fourier series, Fourier transform, wave equations, and heat equations*, are being taught in a computer laboratory setting. As more and more interactive laboratories being developed, this percentage is expected to increase in the future semesters.

In the next section we are going to tour one of the interactive laboratories in *Applied Mathematics* – Wave Equations. Although only one example of boundary value problems will be shown here, the interactive texts are programmed in such a way that they can solve any exercise problem in the related sections of O'Neil's text book *Advanced Engineering Mathematics.*

3. A Tour of An Interactive Lesson

Let us now have a tour of one of these laboratory sessions with our students. Today's lesson is the solution of wave equations by separation of variables.

After the students enacted the interactive textbooks on a computer, the topic of the laboratory are clearly displayed on the screen.

***************** *WAVE EQUATIONS******************

Moving down the screen the students can see how the equation is derived and how separation of variables are used to solve boundary-value problems. There is also a warning that this method may fail if the equation of the boundary conditions are not of right form.

Now students have an opportunity to explore an example. After students follow the instructions on the screen to input the boundary and initial conditions, the screen shows

Therefore, our boundary value problem is

$$\frac{\partial^2}{\partial t^2}y(t,x) = a^2\left(\frac{\partial^2}{\partial x^2}y(t,x)\right)$$
$$y(0,t)=0,\ y(L,t)=0$$
$$y(x,0)=f(x),\ \frac{\partial}{\partial t}y(x,t)=0,\ at\ t=0$$
$$where,\ f(x)=x,\ if,\ 0\le x, x\le \tfrac{1}{2}L$$
$$= L-x,\ if,\ \frac{1}{2}L\le x, x\le L$$

Please verify, and make sure that all the inputs are correct.
Do you want to make some changes?

`Please answer:  yes;  OR  no;`

Students have a chance to correct their input mistakes. If they choose to continue, the following message appears on the screen

Attempt a solution of the form:

$$y(x,t) = X(x)T(t)$$

After substitution and introduction of the parameter λ, the screen shows

$$\frac{\frac{\partial^2}{\partial x^2}X(x)}{X(x)} = -\lambda, \ \mathit{and} \ \frac{\frac{\partial^2}{\partial t^2}T(t)}{a^2T(t)} = -\lambda$$

Students have two things to do. First, they need determine the boundary condition $X(0)$ and $X(L)$. The interactive text asks on the screen:

The boundary value $y(0,t) = 0$ *implies that* $X(0)T(t) = 0$,
What is the value of $X(0)$*?*

`Please enter your answer.`

Students must give the correct answer 0. Otherwise same question will be asked again until the correct answer is given. The second thing students need to determine is the sign of λ. Many students may give correct answer, but few are able to tell why. Therefore, the following questions are designed:

The separation constant λ *must be positive. Otherwise,*
1. the boundary value problem will have no real solution for $X(x)$
2. $X(x)$ *will be identically zero.*
3. mathematically it makes no sense.

`Select your answer by entering  1;  2;  or  3;`

A student need to have profound understanding to make the correct choice (2). This will improve their problem solving ability when a similar situation rises.

By now students have a boundary value problem for $X(x)$:

$$\frac{\partial^2}{\partial x^2}X(x) + k^2X(x) = 0, \ X(0) = 0, \ X(L) = 0$$

where $k^2 = \lambda$. There is no need to let students repeat those standard but lengthy computations they learned in ordinary differential equations. They can simply input a Maple command to solve this differential equation.

Yes, the general solution is $X(x) = \_C1\cos(kx) + \_C2\sin(kx)$

The interactive text also substitutes 0 for x, and uses the boundary condition $X(0) = 0$ to determine $\_C1 = 0$. Thus, $X(x) = \_C2\sin(kx)$. Then the following question appears on the screen.

The boundary condition $X(L)=0$ yields $0=\_C2\sin(kL)$
It looks like that we have two ways to make this equality true:

$$\textit{We can let} \quad 1.\ \_C2=0 \quad \textit{or} \quad 2.\ k=\frac{n\pi}{L}$$

But only one of them is useful to us. Which one?

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Please select   1;   or   2;
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This question is designed to train students' ability of analyzing and solving problems. If students choose the wrong answer (1), they are required to go back and think again. Otherwise, a compliment appears on the screen

Good choice! Indeed, $\_C2=0$ will make $X(x)$ identically 0.

and a summary is given

$$\textit{Therefore, } \lambda=\frac{n^2\pi^2}{L^2},\ n=1,2,3,\ldots,\ \textit{and}$$
$$X(x)=\_C2\sin\left(\frac{n\pi x}{L}\right),n=1,2,3,\ldots$$

Through similar interactive activities, students are able to solve the initial value problem for $T(t)$. After these activities the general solution will be displayed on the screen.

The general solution of the problem is the infinite superposition:

$$y(x,t)=\sum_{n=1}^{\infty} An\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi a x}{L}\right)$$

The next task waiting for students is to determine the coefficients An.

The initial condition $y(x,0)=f(x)$ requires that

$$f(x)=\sum_{n=1}^{\infty} An\sin\left(\frac{n\pi x}{L}\right)$$

Can you recognize what An are in this equation?

Again students have to make a choice among the coefficients of Fourier sine series, Fourier cosine series, and regular Fourier series. Many students need to refresh their memory about these concepts and the formula to find them. The notebook is always ready to help. If students input a question mark "?" at prompt, a review of useful concepts and formulas will appear on the screen. For example,

$>$ hint

The Fourier sine expansion of $f(x)$ defined on $[0,L]$ is

$$\sum_{n=1}^{\infty} b_n\sin\left(\frac{n\pi x}{L}\right)$$
$$\textit{where } b_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

A correct answer is rewarded by relief of lengthy computations

$$\textit{Yes, An are the coefficients of the Fourier sine expansion of } f(x)$$

$$\textit{Therefore, } An = 2\frac{\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx}{L}$$

$$\textit{or, } An = 2\frac{\int_0^{L/2} x\sin\left(\frac{n\pi x}{L}\right)dx + \int_{L/2}^{L}(L-x)\sin\left(\frac{n\pi x}{L}\right)dx}{L}$$

Finally students can enjoy something that they never can do in a chalk and board classroom – an animation of the string vibration from which the given wave equation is derived.

$$\textit{The final solution is}$$

$$y(x,t) = \sum_{n=1}^{\infty} 4\frac{L\sin\left(\frac{1}{2}n\pi\right)\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{n\pi at}{L}\right)}{n^2\pi^2}$$

(Sorry, the LaTex file is not able to print the animation.

Please contact the author for a Maple worksheet.)

Fig. 1 Vibrations of an elastic string

Unfortunately we are not able to print the vivid animation of the vibration that are displayed on a computer screen. The above figure only shows a few positions of the vibration at different time t.

What are the benefits of such a technology-oriented curriculum? This is the question that we are trying to probe in the next section.

3. Impact on Students' Learning

The interactive texts provide a brand new curriculum for the *Applied Mathematics* course which emphasizes the utility of the technology and promotes conceptual understanding. This new environment has improved students' learning in the following aspects.

- The students are invited to become active learners, rather than passive recipients of lectures. Students, not professors, are more actively involved in seeking solutions to the questions and problems. As a result, they are more motivated and enthusiastic in participating in their learning process.

- The interactive texts can present the concepts and methods more clearly and vividly through graphical animation and realistic models. Students are able to visualize concepts and understand them better with this new approach than with anything tried before.

- The interactive texts also reduce the computation and manipulation chores which so many students perceive as the essence of doing mathematics. They now can focus on the critical thinking stage of problem solving like how to use mathematics to find a model for a problem, and how to use technology to answer a variety of interesting and important questions.

- Students will learn the use of technology in performing data analysis. The computer can plot the data in a graph, generate tables, perform mundane calculations, and allow for model testing. In short, the computer removes computational burdens, allowing the student to work on more essential contents.

- An important learning component of interactive laboratory is the small group work on laboratories and in-class projects. Students learn the importance of sharing ideas, brainstorming, and communicating ideas clearly to others. They are given the opportunity to share multiple approaches of analyzing and solving problems. Embedded exercises in the text further promote student cooperation.

References

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