COFFEE, TEA, OR NOT?
A MODEL BASED ON NEWTON'S LAW OF COOLING

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I began using this activity after a discussion with Carl Swenson of Seattle University during a summer Sloan workshop at Dartmouth College. I have found this classroom project to be highly successful in a variety of settings with appropriate modifications. First, a brief description of the activity be given. Next an example will be presented. My concluding remarks will refer to the versatility of the activity.

Upon entering the classroom with a cup of hot tea, I ask the students to help me with a problem. I explain that I frequently bring a cup of hot tea to class but usually I become so busy in class that I do not have a chance to drink the tea until the last part of the class. I then suggest that we collect some data about temperature as a function of time. I have a couple of student volunteers take temperature readings from the cup of tea every five minutes. At the end of the class period the temperature readings are written on the board and the students are asked to think about how they would fit a curve to the data. At the next class meeting I have the students break up in groups and use graphing calculators (or if available the students can use a computer algebra system such as MAPLE) to aid in their investigations.

Let's look a sample set of data:

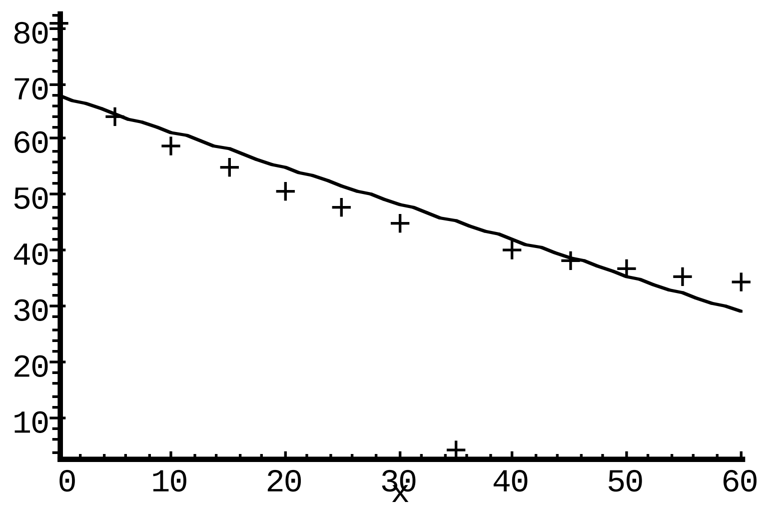
Time	Temperature (C^0)
0	81
5	64
10	59
15	55
20	50.5
25	48
30	45
35	42.5
40	40
45	38
50	37
55	35.5
60	34.5

Using TI-82 calculators and a variety of curve fitting options the students obtained the following results:

Using the linear regression model students obtained the curve:

$$y = -.645x + 67.8$$

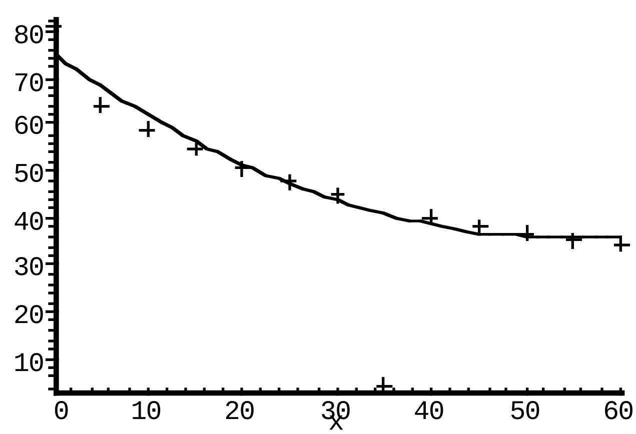
The graph of this line and the original data is given below:



When asked if this looks like an accurate model for the cooling of the tea, a few students quickly point out that as time proceeds the temperature does not tend toward negative infinity but rather the temperature should approach room temperature, in this case 23 ^{0}C. Next they try a quadratic plot, obtaining the results:

$$y = .013x^2 - 1.43x + 75$$

Once again a plot of this curve versus the data exhibits an incorrect asymptotic behavior.



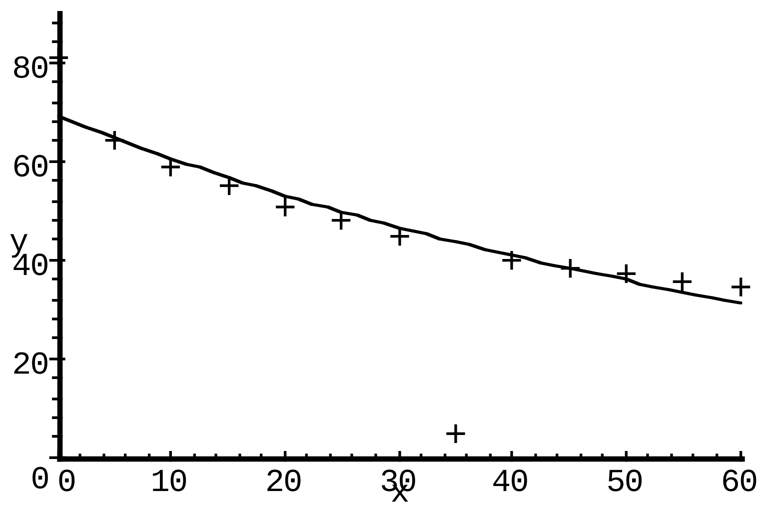
Similarly using a cubic regression option on their calculator produces the equation,

$$y = -.00345x^3 + .0441x^2 - 2.14x + 77.8$$

At this point the students are convinced that a polynomial function is not an appropriate model, at least not over a large time period. Several students almost simultaneously recognize that the curve looks like a decaying exponential function. Since their TI-82 calculators have a menu item listed as ExpReg which fits a curve of the form, $y = a*b^x$ to the data, the students try this option and use the resulting function ,

$$y = 68.9*(.987)^x$$

to plot the data and this function on the same axis.



Now it is for them to decide how they might arrive at a curve to fit the data without the use of the statistics menus on the calculators. In fact, for most of the students these keys are "black boxes" because they have not taken a course in statistics or numerical analysis. Because they have observed that the temperature decays to room temperature rather than zero, in our example they begin to experiment with a curve of the form:

$$y = ae^{-kx} + 23$$

Next they use the initial temperature to determine the value a=58. then they may use a variety of strategies to estimate an appropriate estimate for b. They may use a single data point to solve for k or they may take an average of values of k based on using several data points. For example, using that the temperature at 30 minutes is 45 ^{0}C in the equation above will yield k= .032. The graph below illustrates the approximation properties of this curve to the data.

This activity is successful and appropriate in a variety of settings. I have used it in College Algebra, Calculus I, Mathematical Modeling, and Differential Equations. Naturally the depth of exploration of this problem will vary depending upon the class. In differential equations the students would be asked to consider the rates of change in the temperature and they actually end up deriving Newton's Law of Cooling, $\frac{dy}{dx} = -k(x - r)$, where y represents the temperature at time x, in a setting with temperature r.

The students are asked to decide whether or not this is a good choice for a function to represent the relationship between temperature and time. Very quickly students point out that as time proceeds the temperature settles down to room temperature rather than negative infinity as the linear model predicts.