TWELVE PRACTICAL IDEAS FOR THE USE OF THE TI-82 IN THE SECOND-TERM CALCULUS COURSE

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The TI-82 calculator can be used to help motivate and enliven many of the fundamental ideas in the second-term calculus course. This can be done easily and naturally, without having to abandon your present text or adopt a radically different teaching style. Several simple examples of calculator use are presented in this paper, in the hope that those who have not yet begun teaching with a calculator will find that it is indeed both easy and natural. This paper is a companion to the author's *Twelve Practical Ideas for the use of the TI-82 in the First-Term Calculus Course*, presented at the Sixth Annual ICTCM.

At John Carroll University, the TI-82 has been adopted as the recommended calculator for use in all calculus courses. Although the calculator is the only form of technology used in the first-semester courses, computer software such as Derive is integrated into the second and third-semester courses, for symbolic manipulation and graphing beyond the capabilities of the TI-82. The text in use is Varberg & Purcell, 6th edition. Although this text is amenable to the use of a graphics calculator as a supplement, the calculator is not essential to the text. Likewise, it is the author's conviction that the TI-82 could be used equally effectively with any of a long list of texts. Accordingly, the ideas presented in this paper could be used in almost any calculus teaching environment, to ease the computational burden and focus the students' attention on the *ideas* of calculus rather than on calculation. The specific ideas presented here have all been used by the author in class, and are a portion of a document made available to colleagues in the author's department who expressed some uncertainty as to how the calculator might be used in their courses. Before presenting these ideas, a few general suggestions are offered.

General advice

a) Advise students to *bring their calculators to class every day.*

b) Make the calculator a natural tool in the problem-solving process, rather than an "add-on." Allow and encourage students to use it on *any problem,* wherever it is useful, rather than assigning special "calculator problems." By your use of the calculator on in-class examples, lead the students to recognize when it is useful and when it is unnecessary.

c) Have students use their calculators fairly frequently in class. If you have a TI ViewScreen, resist the temptation to turn to it too quickly, as this will discourage students from using their own calculators in class. Instead, ask the students to use their calculators to do something–and then, after 15 seconds or so, follow up with the same thing on the ViewScreen, so they can check their work for correctness.

Idea 1

In the sections on applications of the integral, the *concept* of the integral and its application can be emphasized by shifting the burden of *calculation* to the TI-82. For simple problems of area under a curve, use 2nd CALC 7 ($\int f(x)dx$) to have the students find integrals. This requires tracing the curve to mark the left and right endpoints, and shows the area as a shaded region. For the area of a region between two functions of x, draw the graphs and use 2nd CALC INTERSECT to find the points of intersection. After finding the first point of intersection, store it in variable A. Find the second point of intersection and store it in B. After setting up the integral for the area, evaluate it using MATH 9; for instance, fnInt($\mathrm{Y}_1 - \mathrm{Y}_2$,X,A,B).

Idea 2

To introduce the natural log function, assign each student one of the following numbers: .1, .2, ..., .9, 1, 1.2, 1.4, ..., 3.8, 4.0. Then have them compute, on their TI-82's, the value of fnInt(1/X,X,1,A), where A represents their personal number. Record these values in a table on the board. Have the students plot these points by hand, and attempt to draw the graph.

Next, have them enter the formula $\mathrm{Y}_1 =$ fnInt(1/T,T,1,X,2), and graph in the window $[0, 9.4] \times [-3.1, 3.1]$, which has the correct aspect ratio and will show the essential features of the graph. Why does the graph take so long (about 40 seconds)? Use TRACE to verify that the y-coordinates are the same as those previously calculated. (With the window suggested above, all x-values used in the hand-drawn graph will be visited by the trace.)

Next introduce the LN key on the calculator, and have the students graph $\mathrm{Y}_2 = \ln(\mathrm{X})$ along with the graph of Y_1. Is there any difference? Why does the graph of Y_2 take so much less time? (Note: If you are using a ViewScreen, come to class with a cut-out of an overhead transparency which will fit exactly over the view screen's panel. Using a transparency marker, trace the graph of Y_1 onto the transparency. You can then deactivate this function, draw the graph of $\mathrm{Y}_2 = \ln(\mathrm{X})$, and have the class observe that it matches the previous graph.)

Having established graphically that $\displaystyle\int_1^x \frac{1}{t}\,dt$ is the same as the calculator's LN function, have the students use LN instead of the numerical integration, in order to save time. Enter $\mathrm{Y}_1 = \ln \mathrm{X}$ and $\mathrm{Y}_2 =$ nDeriv(Y_1,X,X). Deactivate Y_1 and graph Y_2, using ZOOM ZDecimal. Do you recognize the graph of the resulting function, $\displaystyle\frac{\mathrm{d}}{\mathrm{dx}}\int_1^x \frac{1}{t}\,dt$? For students with good memories, this will not be a surprise.

Idea 3

The reflection of the graph of a function into the graph of its inverse can be illustrated easily on the TI-82. For example, graph $y_1 = x^3 + 2$, $y_2 = x$ and $y_3 = \sqrt[3]{x-2}$ using ZOOM ZDecimal, and observe the reflection property. Have the students try this again with another function, say $y_1 = \frac{x}{1-x}$ and its inverse, $y_3 = \frac{x}{1+x}$.

The inverse function theorem can be illustrated easily with graphs such as these. For example, using $f(x) = y_1 = x^3 + 2$ and its inverse $f^{-1}(x) = y_3 = \sqrt[3]{x-2}$, we will find the derivative of f^{-1} at $x = 1$. Use 2nd CALC 6 (dy/dx) and press the cursor-up key to move onto the curve Y_3—that is, f^{-1}. Move along this curve to $x = 1$, and press ENTER to get the derivative, $\frac{dy}{dx} = .3333334$ (that is, 1/3). Now use 2nd CALC 6 again. Since $f^{-1}(1) = -1$, move along curve Y_1 to $x = -1$, and press ENTER. The TI-82 shows that the derivative here is 3.000001 (that is, 3). Observe that these two derivatives are indeed reciprocals.

Idea 4

In your book's section on growth and decay models, some problems will undoubtedly call for determining the time at which the growing or decaying quantity reaches a certain value. As an adjunct to the traditional symbolic solution of this type of problem, a graphic approach can be used to solve a growth model for t. The calculator can also be used to illustrate the graphs of various growth and decay models.

Some books introduce $\lim_{h \to 0}(1+h)^{\frac{1}{h}}$ in this section. Wherever it is introduced, the TABLE feature of the TI-82 makes it easy for students to be able to *discover* its value.

Idea 5

The exponential function has many applications in addition to growth and decay. For instance, the normal probability density function, defined by $y = \dfrac{1}{\sigma\sqrt{2\pi}} e^{[\frac{-1}{2}(\frac{x-\mu}{\sigma})^2]}$, is used very frequently in probability and statistics. The standard normal density ($\mu = 0$, $\sigma = 1$) can be graphed easily on the TI-82, and probabilities can be calculated with 2nd CALC 7. An appropriate window is [-2.35,2.35] $\times$ [-.5,.5]. With this window, integrals can be calculated using lower and upper limits which are multiples of .05

Idea 6

When first introducing the inverse trig functions, use student-drawn graphs of the trig functions to show the restriction of the domain needed to define the inverse function. Alternatively, if you have a ViewScreen, use an overhead slide cut-out and a transparency marker to highlight the appropriate portion of the graph where the function is invertible.

Idea 7

To introduce the derivatives of the inverse trig functions, have the students enter $Y_1 =$ nDeriv($\tan^{-1}$X,X,X), and graph using ZOOM ZDecimal. Does this look like a trigonometric function? Compare this graph to that of $Y_2 = 1/(1+X^2)$. (You may wish to have students TRACE, and switch between Y_1 and Y_2 at a few points, to verify that both graphs have indeed been drawn.) **Question:** Do you think these two functions are really identical, or just very similar? Remembering that nDeriv gives only a numerical approximation, we might conjecture that $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$. Does it seem surprising that the derivative of an inverse trig function should be an algebraic function? This discovery may pique the students' interest in *why* this happens, and thereby make them more accepting of the proofs of such formulas.

Idea 8

After introducing L'Hôpital's rule, have students check their answers by drawing the graph of the function in an appropriate window. (Many students will benefit from such continued reminders that the limit concept is a natural one, which addresses questions concerning the appearance of a graph.)

Idea 9

The best linear approximation of f in a neighborhood of the point $((a, f(a))$ is, of course, the tangent line. To drive home this point, have the students graph a function such as Y_1 = ln(X) using ZOOM ZDecimal. By hand, compute the tangent line, Y_2 =X-1, and graph this also. Next, set the zoom factors to 4, trace to (1,0), and zoom in several times. After about 4 zooms, there should be no visible difference between f and its tangent line (in the neighborhood defined by the current window).

How small is this window? Can we find a slightly more complex function, say a quadratic, for which the approximation will also be good, but in a larger window? Have the students graph the quadratic Y_3 = -.5X^2 + 2X - 1.5 (leaving Y_1 and Y_2 active), using ZOOM ZDecimal. Then set the window manually to $[.5, 1.5] \times [-.5, .5]$, and it should be clear that this particular quadratic function does a better job of approximating f in this window, than does the linear function.

Some natural questions:

- How did we come up with the formula to use for the quadratic function?
- Can we do an even better job? (In a global sense, the quadratic is not all that good an approximation of f.)
- How do we measure "how good" an approximation to f is?

Hopefully, this approach will serve to motivate the study of Taylor polynomials and even (shudder) the remainder!

Idea 10

Newton's method can be computed easily on a TI-82, even without programming. (Many students have an aversion to programming!) For a non-programming approach, enter the formula for $f(x)$ in Y_1, and enter nDeriv(Y1,X,X) in Y_2. In Y_3, enter X-Y_1/Y_2. Quit the Y= menu, enter your initial guess and store it in X. Then select Y_3 from the Y-VARS Function menu, and press Enter. (This will evaluate the next approximation.) Next store this in X, and then use 2nd Entry *twice* to recall Y_3, and press Enter to get the next approximation. Repeat these two steps the desired number of times.

While you are on the topic of Newton's method, you might remind students of the TI-82's built-in root-finding algorithm in 2nd CALC ROOT. While it is clear from the way the TI-82's algorithm works that it is not Newton's method, the students will at least have a much better appreciation of the general sort of logic which the calculator actually does use.

Idea 11

The TI-82 excels at drawing graphs of polar equations. In the MODE menu, change the graph type (on the fourth line) to "Pol." Enter an equation in the Y= menu, which will now automatically accept polar equations. (The X,T,θ key now gives θ.) Select a window and Graph, being sure that the θmin and θmax values are appropriate for your problem. Also be sure that θstep is small enough to give appropriate detail in the graph, but not so small that the graph will take forever. A θstep of $\frac{\pi}{24}$ will assure that a trace of the graph will visit the points associated with the angles which you would use if you were plotting the curve by hand: $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, etc.

Idea 12

The TI-82 is also an excellent tool for graphing parametric curves in the plane. In the MODE menu, select "Par" on the fourth line for parametric graphing mode. The Y= menu then accepts *pairs* of parametric equations, and the X,T,θ key gives the symbol T. Be aware that you may need to change Tmin and Tmax in the WINDOW menu, and check to be sure that the value of Tstep is appropriate. You may wish to calculate the arc length of a few parametric curves. Although innocent-looking arc length problems can often yield integrals which are symbolically intractable, the TI-82 can quickly produce approximations of such integrals.

In conclusion, the ideas presented above can be incorporated into any second-term calculus course, with relatively little additional effort on the part of the instructor. There is no claim that these ideas are either original or the best possible applications of the TI-82. They are simply some of the ways in which the author has found the TI-82 to be a natural and useful addition to an otherwise standard course.