GRAPHING CALCULATOR USE IN CALCULUS I.

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Recent breakthroughs in computing technology, interactive computer and calculator graphing, are becoming part of the mathematics instruction at the secondary and college level. Research shows that these technologies can be used successfully in mathematics education. They can open new horizons and they can revolutionize what and how we teach in mathematics [6, 7, 10, 12]. However, since these opportunities became available only recently, this is still a new, unexpected, unstudied, and unpredictable area in mathematics education, where there is a great need for further research [11].

Learning by discovery, on the other hand, has been one of the most studied and most controversial issues in mathematics education [2, 5]. A great number of arguments have been given both for and against; a great number of experiments have been conducted. However, research results are still conflictive and inconclusive [2, 4, 5, 8, 9]. There were numerous calls for further research on the effectiveness of the discovery style teaching [1, 2, 13], particularly for research under regular classroom circumstances [3], dealing "with large segments of instructional material and not merely with short-term problem-solving exercises in the laboratory" ([1], p. 561). The development of interactive computer and calculator graphing gives new momentum to the discovery movement since it facilitates student experimentation and discovery.

This paper describes a three-group experimental study conducted in an introductory university differential calculus course at The Ohio State University with the following design.

<u>Group 1:</u> Use of graphing calculators
+ (guided) discovery approach

<u>Group 2:</u> Use of graphing calculators
without discovery

<u>Group 3:</u> No graphing calculators,
no discovery
(traditional instruction)

The two major objectives of the study were to verify that students *can* discover a significant portion of differential calculus and to investigate the effects of the use/non-use of graphing calculators and the instructional technique (lecture/discussion or guided discovery style teaching) on student achievement (conceptual understanding, ability to transfer what has been learned to a different but related situation, computational skills, short and long term retention), students' time spent on course, the extent to which students worked with classmates outside of class, and the extent to which students actually discovered the new material.

In the discovery section, part of the new material was covered using worksheets, where a chain of questions/problems led to the new concept, relationship, or technique. Students worked in groups, pairs or individually. The worksheets were supplemented with hint-sheets and solution-sheets (the latter with complete solutions, the former with some hints only). If students needed help they could look at these *cheat-sheets*, but only one line at a time, and then they were to continue on their own. They could also ask their classmates and/or the instructor. They were to check their answers after each problem using the *solution-sheets* and correct them if necessary. Obviously, students cannot discover everything mankind has discovered in 2000 years (especially not in 50 x 48 minutes); thus (the larger) part of the material was covered in the traditional discussion format.

According to the questionnaire students completed after the final exam, students found the answer on their own to 47% of those questions on the worksheets where the answer was not previously known to them. They found the answer to an additional 22% of the questions with hints from the *hint-sheets*, from classmates or the instructor. Over 75% of the students found the answer to the majority of such questions with or without hint. 88% of the students suggested that some classtime (in average 30%) be spent on discovery style teaching. This shows that discovery style teaching is a viable alternative to traditional teaching for at least part of the new material.

The groups were compared before and after instruction. No statistically significant differences were found on the computational, conceptual, and transfer skills parts of the pretest. No statistically significant differences were found on the following background variables either: placement level, the year in which students took the placement test, their precalculus grade and the year in which they took precalculus.

Analyses of covariance were used for student achievement comparisons. The scores on the corresponding subtest of the pretest served as covariates. Students' time spent on the course and the extent to which students worked with their classmates outside of class were also compared. Statistically significant differences were not found between the groups on any of these variables. No instructional method proved superior to the others on comparison.

A sample hint-sheet follows. The corresponding worksheet is identical, but without the hints written in Zapf Chancery font (imitating handwriting) and without the graph. The corresponding solution-sheet contains all the written text in Times font in the hint-sheet and complete solutions to all problems.

```
PrgmA:NEWTON01
:Input X
:X-Y1/NDeriv(Y1,.00001) → X
:Disp X
```

Figure 1.
TI-81 program for Newton's method. Hit ENTER and then X | T for next approximation

```
PrgmB:NEWTON02
:Input X
:Lbl 1
:X-Y1/NDeriv(Y1,.00001) →X
:Disp X
:If abs (Y1/NDeriv(Y1,.00001))<.000000000001
:Stop
:Go to 1
```

Figure 2.
TI-81 program for Newton's method with 10^-12 accuracy (10 digits displayed).

3.8 Newton's Method for Approximating Solutions of Equations
HINT-SHEET

Intro. It is often necessary to give approximate solutions to equations. A general fifth or higher degree equation *cannot* be solved algebraically, and the same is true about most equations involving both trig and algebraic terms, eg., sin $x = x^2$. Other equations *can* be solved algebraically, but the solution is so long that a faster approximate solution is often preferred. (Eg., all third and fourth degree equations *can* be solved algebraically, but it is certainly a cese of *cruel and unusual punishment*.) In this section you will learn one of the fastest and most widely used approximation technique, the Newton-Raphson method.

1. Find the x-coordinate of the point where the curve $y = x^3 - x$ crosses the horizontal line $y = 1$.

Solving this problem is equivalent to solving the equation

$x^3 - x =$ (for the intersection point) or _______________ = 0.

The number of solutions and a rough estimate can be given by graphing calculator. In fact, a solution of any desired degree of accuracy (up to machine-precision, 8 - 10 significant digits for most) can be given by graphing calculator (*zoom - zoom*), but the graphing calculator alone is not the fastest way when high precision is required.

This equation has __ __ solution(s). Reasons (give a rough sketch + a few sentences):

Graph $y = x^3 - x - 1$

If x is a solution, then $x^3 - x - 1 =$ __ for the x value.

So that point is on the __ axis.

Did you include your argument why you can be sure there are no additional solutions outside of your viewing rectangle (screen)? Make sure to check the *cheat-sheet* to see if your argument is complete.

Finding the solution of our equation is equivalent to finding where the graph of f(x) = *$x^3 - x - 1$ crosses the* ______________. The graphing calculator shows that the solution is close to the integer x_0 = *1*, this will be our first (or 0[th]) approximation, and we will improve our approximation step by step.

Sketch the graph of our function f(x) = *$x^3 - x - 1$* around its x intercept, on the interval [1,2], and add everything we are talking about as we go along.

1.1 First, we can approximate f(x) around x_0 =1 by ___*(See 3.7)*___________. Now, if our FUNction can be approximated by this line, then its x intercept (the solution of our equation) can be approximated by _____*the x intercept of the tangent line*_________. The slope of this line is m = _*f '( )*_ = ______.

Therefore, an equation of this line is ______________.

The x intercept of this line is x_1 = ___*(Use point-slope formula)* .

This will be our next (improved) approximation.

1.2 To improve our approximation, x_1, we can approximate our function around x_1 by *its tangent line at x =* _____, and proceed exactly the same way as in **1.1.** Use as many decimals as your calculator can give you.

Slope of tangent line, m =

Equation of tangent line:

x intercept of tangent line:

The improved approximation is x_2 =__________________.

1.3 You can keep going if you wish (but don't spend too much time on it) or just view the next four approximations. Be prepared to be impressed!

The results of applying Newton's method to $x^3 - x - 1 = 0$ with $x_0 = 1$:

x_1 = 1.5
x_2 = 1.347826087
x_3 = 1.325200399
x_4 = 1.324718174
x_5 = 1.324717957
x_6 = 1.324717957

How many correct decimals and how many significant digits do you think our last estimate, x_6, has?
Number of correct decimals: __________. *(i.e. how many digits after the decimal point are correct)*
Number of significant digits: ________. *(i.e. how many digits altogether are correct)*

2. The Formula for Newton's Method

This method works nicely, but it would be too long to go through this process each and every time you need to solve an equation. Wouldn't it be nicer to derive a formula into which we can just plug in numbers to get the next approximation? (Then we can write a short, one line program that gives the next approximation using the formula. Or, better yet, write a short, few liner to give the sequence of approximations. A possible solution - in BASIC - is in your book, see pp. 206 - 208, Ex 3, 4. The program can be much shorter if you want less features.)

So, now let's derive this formula. It will be easier than you think. We'll do exactly the same as in **1.**, but with parameters (letters), rather than numbers. Let our equation be f(x) = 0, the starting estimate x_0. Compute x_1. Turn back to the previous page if needed.

$y - f(x_0) = f'(x_0)(x - x_0)$

x_1 = ______________.

Guess what's next? Go for x_2. You might want to think a little if you can get this formula without any slave work (computation), just by taking a look at the previous formula (x_1 = ...).

x_2 is obtained (computed) from x_1, the same way as x_1 is computed from x_0.
This should be reflected in the formula.

x_2 = ____________________.

In general, x_{n+1} = _______________.

3. To see how this formula works, do our first problem with this formula and compare each step with our previous results (1.).

$x_0 = 1$ $\qquad$ $f(x) = x^3 - x - 1$ $\qquad$ $f'(x)$ = ________________

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = \text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}$$

x_3 = ________________________________

4. Can you summarize how you solve an equation of the form $f(x) = 0$ using Newton's method?

1. *Find initial approximation, x_0.* ____

2. *Use x_0 to get a 2^{nd} (better) approximation by the formula.* ________________________

3. ____

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