

MESHING MEANINGFUL STUDENT PROJECTS INTO A FAIRLY TRADITIONAL CALCULUS COURSE

Dr. Kemble Yates
Southern Oregon State College
1250 Siskiyou Blvd.
Ashland, OR 97520
kyates@wpo.sosc.osshe.edu

How will I change how I teach calculus? That was the question I faced at the beginning of Winter quarter, 1993. As a graduate student at a large research university, I had seen first hand the failed model of lecture/common exams. Watching 50-75% of my students get a D, F, or W convinced me that the old way does not work. My experience of teaching for five years at a 4 year liberal arts college was only slightly better--smaller classes and writing my own exams helped but did not dramatically improve my teaching effectiveness. I had read for some time about the paradigm of a lean and lively calculus: more technology, less lecture, fewer but richer topics, and cooperative learning. As our department had just mandated graphing calculators for all precalculus and calculus courses, and several colleagues were engaged in group work and student projects in their courses, I resolved to try *something*. And so I decided to do a little bit of everything! I structured my second term calculus class to have some lecture, some discussion, some group work, some use of graphing calculators, and some student projects. On the whole, I was pleased with the results--especially the student projects. But I did learn from the experience and have some thoughts on how to do this better the next time.

There were many constraints on how and what I could do. I was teaching the middle course of a three course sequence. I had not taught the first course nor was I scheduled to teach the following course. Therefore I was obliged to "plug in" to the syllabus and cover a certain amount of material. We generally start Calculus II with integration, cover u-substitution, explore some applications of integration, derive the logarithm and exponential functions, and finish with hyperbolic and inverse trig functions. The quarter system itself is limiting--ten weeks plus a finals week. The text was already chosen,¹ and had the virtue of being "technology aware." There were many graphing calculator exercises and discussions, plus a nice appendix with programs for the TI-81, Casio, HP-28S, and HP-48SX. On the down side, many students found its exposition murkier than our prior text of many years.² The students were required to buy a graphing calculator and the department recommended the TI-81 at that time. We were in the first year both of using this text and of requiring graphing calculators. Into this environment, I launched my new plan.

Requiring three student projects formed the basis of my new approach. I assigned the same project to all of the students, randomly chose pairs to work together, and gave them 10 calendar days to complete their work. The handout for the project outlined the question to be pursued,

¹Calculus, 3rd ed., Berkey and Blanchard; Harcourt, Brace, and Janovich, 1992.

²Calculus, 2nd and 3rd eds., Anton; Wiley, 1990.

the tasks to be performed, and all the items to be turned in³. I gave each student of the pair the same grade, and all three projects counted for a total of 10% of the course grade. The projects did cost some class time: I allotted 10 minutes to introduce each project, and I also gave the students 20 minutes of class time to get started, i.e. to discuss their schedules and plan times to meet. But the emphasis was on time spent *outside* of class. I did notice a significant increase in appointments and office hours utilization while a project was underway. But the regular class continued--I covered new material, assigned new homework, and administered the usual weekly quizzes.

The first project involved acquainting the students with programming their graphing calculator to perform the midpoint method of numerical integration. I took a full class day prior to assigning this project and introduced programming on the TI-81, culminating with the students working in groups to type in a Newton-Raphson routine. I had several reasons for choosing numerical integration for the first project. Integration is the first topic scheduled for Calculus II and I believed the midpoint method nicely reenforced the discussion of Riemann Sums students were working on in homework. Another reason was pragmatic: the TI-81 does not have a numerical integration routine, and I wanted the students to have one.⁴ A third rationale for this project was, candidly, to help justify the requirement of purchasing a graphing calculator. Since students had already learned (in theory) all about the graphing capabilities of their calculators, I wanted them to stretch their calculator knowledge to programming. While many teams completed the midpoint program requirement by modifying a program in the back of the book, many actually wrote their own routines. In either event, they were forced to learn how to enter a program into their calculator.

After getting a working program, the teams had to look in detail at two problems. The first was to numerically approximate π using

$$\int_0^2 \sqrt{4 - x^2} dx .$$

Their instruction was to find the smallest number of rectangles needed to approximate π to 4 decimal places. The second question concerned numerically investigating the initial value

problem
$$\frac{dp}{dt} = \frac{100 - 25t}{t^2 - 8t + 16.1} ; p(0) = R ,^5$$

where p represents the population of rabbits. I asked them to find a good window of $\frac{dp}{dt}$

³ Hardcopies of the actual project assignments are available by contacting me by e-mail or regular mail.

⁴I subsequently passed out a TI-81 routine for Simpson's Rule.

⁵This is a modification of problem 59, p. 353, from Berkey and Blanchard.

and t on their graphing calculator, and then make a good sketch on graph paper. They were then asked to determine when the population reaches a maximum. Notice that this question nicely reviews graphical interpretation of a derivative: the maximum population is reached when the derivative is 0, not when the derivative reaches a maximum (which is what many of the students first believe). I then had them evaluate the solution

$$p(t) = R + \int_0^t \frac{dp}{ds} ds ,$$

where R is the sum of the team members' ages. Calculus students are not comfortable with a function defined by an integral. But I felt that giving that definition a tangible context helped them fight through the symbols and ultimately to invest some meaning in them. After evaluating the rabbit population function at numerous points using their midpoint routines, they were asked to determine when the population would go extinct, according to this model. Some interesting numerical questions arise in the construction of $p(t)$. In particular, the wider the interval of integration, the more error in the solution (assuming a constant number of rectangles). Some of my students saw this and correctly diagnosed that more rectangles are needed to accurately compute the integral over a larger interval. And of course, many did not--which led to some interesting and amusing graphs.

The second project consisted of the students choosing any country in the world, and numerically finding its area and geographical center. This project was assigned while we were covering the section on applications of the definite integral. After obtaining a photocopy of a map of their country, the teams had to set up a coordinate system, divide the x-axis into an appropriate number of subintervals, measure heights on each subinterval using a ruler, compute the area using Simpson's Rule, and convert the answer into real area units (typically square miles). Notice that this required them to use Simpson's Rule on a function with tabular values, which contrasts nicely with typical text book applications where a function is defined by a formula. Next, the students had to read in their text about centroids (geographical centers), use Simpson's Rule again to find the geographical center of their country, and plot this point on their maps. Notice this computation necessitated a conversion from their units to degrees of latitude and longitude. Upon finishing their computations, I asked them to compute a percentage error of their findings with values found in an atlas, and to comment on their error. Unfortunately, many had difficulty finding a published geographic center for their countries!

I was pleased and impressed with the students' work on this project. In short, they *enjoyed* doing it. The project served several mathematical goals: cementing Simpson's Rule, performing Simpson's Rule on a function given by a table, reinforcing the integral-as-area concept, and exposing them to an extra application (centroids) which I never covered in class. But in addition, the project stirred student interest and made a nice connection between a "real world" problem and calculus concepts. I was initially concerned that this project was light on technology, but my concerns were unfounded. A few teams saw immediately that the numerical computations are most efficiently performed and displayed in a spreadsheet. One team even wrote a TI-81

program to do the computations.⁶

Applying some simple population models to an actual country formed the third and final project. Once again the teams were asked to choose a country: this time, they needed to find good population data going back at least 100 years if possible. In class, we covered exponential and logarithmic functions, and the exponential growth model. In the project, the students had to explore some of the mathematical details of both the exponential growth and logistic growth models. These models are expressed as initial value problems:

$$\frac{dP}{dt} = kP ; P(0) = P_0 , k > 0$$

and

$$\frac{dP}{dt} = aP - bP^2 ; P(0) = P_0 ; a, b > 0 .$$

The students were exposed to separation of variables to solve these initial value problems, and then had to do some significant derivations to yield formulas for the parameters of the logistic growth model in terms of the data. I also asked them to differentiate the solution to the logistic model and show that it satisfies both the differential equation and the initial condition. After performing the mathematical preliminaries, the students fit exponential and logistic models to their countries. I insisted that they generate at least two different models of each type--one whose parameters were derived from a shorter time interval (e.g. 20 years) and one from a longer time interval (e.g. 100 years). They then had to assess their models and make predictions from them. This was done by testing each model's prediction for the 1990 population and comparing the prediction with a published figure. Finally, the teams had to give each model's prediction for the country's population in the years 2000 and 2050, and to write about which model they felt gave the most believable predictions and which model gave the least believable predictions.

This project was a nice confluence of mathematical ideas, application, modeling, and critical thinking. Exposure to two more initial value problems (their first was in Project 1) helped solidify the interconnection between differentiation, integration, and algebra. Using the exponential and logarithm functions to answer a question of interest--future population growth--motivated the need for truly understanding how these functions work. The graphing calculators were useful in doing the computations and for graphing the resulting solution to the model. In their discussions, some of the teams saw quite deeply into the modeling problem. Many cited the two World Wars' influence on population data, while others saw that some data sets do not lend themselves to a logistic model.⁷

⁶Note that a typical Simpson's routine will not work because it depends on the function being defined by a formula.

⁷Both parameters of the differential equation in the logistic model must be positive, or the resulting model is not biologically realistic. One team tried to find sufficient conditions on the data for both parameters to be positive, which was well beyond the intended scope of the project.

What would I do differently? Most importantly, I would have the projects count for more of the students' grades. Instead of having the projects count for a total of 10%, *each project* should count for at least 10% of the course grade. The students pointed this out to me and they are right: typically, a team would spend 8-12 hours completing a project. I was concerned *a priori* that the projects might dampen individual responsibility, but I am absolutely convinced that did not happen. The peer pressure to "pull your weight" on a group project was tremendous. My choice to randomly select the pairs to work together worked remarkably well. I did pair people with the same brand of calculators together; I think special case pairing, e.g. car pool companions, people with full work schedules, etc., is necessary and fine. But I do think it was valuable on at least the first project to have random pairing: getting the class to know one another and to work with one another is an excellent goal and can be undermined if preexisting friends are allowed to avoid interacting with new people. Giving the teams a few minutes of class time to introduce themselves to each other, compare schedules, and make some plans for meeting outside of class is essential. It is crucial to get the teams started on the right foot and this initial meeting in class really helps them over the initial hurdle of "we haven't done this before." I believe the whole structure of projects would work even better in a semester environment. Having sufficient calendar days to have projects going while attending to other course goals really helps. Another mechanical change I will make in the future is to consciously assign a few less homework problems while the teams are doing their projects. The reality is that time spent on the projects will, at least to some extent, come at the expense of other course requirements. There were a few times when I was not conscientious enough about the length of a given homework assignment relative to the due date of a project. Along these lines, it is necessary to judiciously schedule exams with sufficient spacing after project due dates in order for the students to properly prepare for exams. In writing new projects and modifying the ones presented here, I will try to be more prescriptive in demanding written responses to questions and providing supporting work like graphs. For instance, in Project 3, I expected (but often did not receive) graphs of the various models that the teams had generated. I had explicitly asked for graphs in Project 1 and assumed the teams would naturally include graphs in Project 3. Moral: if you want something, you have to ask for it. There is a delicate balance here. As teachers, we need to be very prescriptive about what questions we want answered and what tasks we want performed, but we must be careful not to then tell the students how we want questions answered and succumb to just "telling them what to do." A final issue for future projects is the obsolescence of the TI-81 calculator. The built-in features of the TI-82 and TI-85 make programming a numerical integration routine into the calculator a much less urgent priority. I would still maintain, however, that there is much to be gained by forcing the students to learn how to program their calculators. But the expanded capability of the newer Texas Instruments machines, as well as the other brand names, means we can assume students have the capability to solve (at least numerically) very challenging root, extrema, and integration problems; this should only broaden and deepen the scope of future projects I assign my students.

In conclusion, I strongly believe these three projects produced an extremely positive change in my Calculus II class relative to my previous classes. Along with having groups work on homework problems periodically through the course, the projects truly fostered a cooperative learning environment. One simple positive measure was that the students all knew each other by the end of the quarter. The projects inspired the teams to dive more deeply into calculus than typical text book homework assignments do. The comments on my teaching evaluations were

overwhelmingly positive regarding the projects. I am also convinced that the group projects made a more effective use of technology in learning than class time activities or text homework. By working with each other outside of class time, students learned how to use their calculators far more efficiently than I could ever teach them in class. Furthermore, by integrating the calculator into the first project, I believe the students were given an excellent opportunity to see the proper role of a graphing calculator: it is an excellent means toward an end, but ultimately it is just one of many tools in formulating, solving, and interpreting mathematical problems.