COMPUTERS IN MATHEMATICS EDUCATION - AN EXPERIENCE

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The first formal course on Computers in Education to be created in a portuguese university, was in 1987 at the Mathematics Department of the University of Coimbra and has the name "Computers in Mathematics Education". In our Department we prepare school teachers (for grades 5-12) since 1972. They have the first two years in common with pure and applied mathematics students, in the two following years they have more courses on some aspects of Mathematics, on Mathematics Education and on Psychology; in the fifth and last year they have a stage in a secondary school where they are supervised by a school teacher and a university professor.

A national project was created in Portugal by the Ministry of Education (the Minerva project) in order to introduce computers in primary, basic and secondary schools (grades 1-12). From the beginning we saw that the emphasis was too much in hardware and programming languages (including Basic, Pascal and also Logo) and not much on the particular aspects and uses of each scientific discipline. And in fact this national project ended in 1994 and we can say very few teachers use computers in their teaching (we can easily estimate they are less than 5%).

We think that the right way to prepare future school teachers is to put them at ease with what they will teach later. If they would be at ease doing mathematics with the computer than they would be prepared to use it in their normal teaching life.

This was the main reason why we proposed the creation of this course. Fortunately the Mathematics Department accepted it and it is now a compulsory course for future school teachers in the second semester of their fourth year of studies. We have been the two responsible for this course over the years and we would like to discuss briefly how it was conducted.

We have 2 or 3 hours a week of theory and demonstrations and 3 hours a week of practical work in a computer lab. At first, only computers were in this lab; now we also have graphing calculators. In this lab we have now 12 IBM compatible computers that were bought over the years (and so are of different models, some without hard disk). Some of the classes are given in another lab with 8 Macintoshes. We have more or less 60 students each year that are divided into three groups of 20 students working in groups of two for the sessions in the computer lab.

In the theoretical course we discuss what is detailed in the first part of Annex 1. This includes discussions of what computers are, what software is, how the computer can be used in the mathematics classroom, how mathematics is taught in a different way with the computer with a bigger involvement from the part of the student, which types of software are available (we make an abstract classification of educational software), and we give examples of how things work (using a portable computer and a data display). We put a lot of emphasis on the connections with the portuguese curriculum being used each year. The new portuguese curriculum encourages the use of problem solving, connections of mathematics with the real world, use of calculators an computers, and the official documents are discussed and are available at the Department's Library. Several papers from portuguese journals are available to the students and we give them a list of references where they can find more information about the topics discussed in the classroom. This list of references is given in Annex 3.

In the practical sessions, students work with different pieces of software some of which are referred to in Annex 1. They experiment the software, they write small reports about the software, they execute small projects or they have to plan small projects. At first students do not feel at ease with the computers. Before that they only worked with terminals of a big computer (they never saw). It is the first time they are in front of the "beast". After the first two weeks of classes students feel enormously at ease with the computer and most of them discuss with interest a lot of aspects that are related with the use of computers in the mathematics classroom.

In the practical sessions there is space for pure exploration where the student can experiment the software at ease, and space for the execution of worksheets written for secondary school students (some of them have already been experimented with students). One of them includes the study of a sequence like

$$x_0 = 0 \qquad x_{n+1} = 1 - k\left(x_n\right)^2$$

with a spreadsheet or the software "MATHPROGRAM" that allows to graph easily the first 200 terms of several sequences at the same time. Another one includes the study of population growth numerically and graphically. Some worksheets are related with statistics, some with geometry.

The assessment is made of two parts each worth 50%. The students must do 4 or 5 small projects (some individually and some in groups of two) during the semester and they have a final examination. A sample examination is given in Annex 2. The projects must be made outside the sessions of practical work. The students can work freely in the computer lab outside these hours. The projects include things like:

"*Make a worksheet for a secondary school student that may lead him to discover numerically and graphically, with the help of the computer, how the limit of the following sequence*

$$u_n = \left(1 + \frac{L}{n}\right)^{Kn}$$

varies with L and K."

These sequences can be defined recursively like:

$$u_0 = K; \quad u_{n+1} = \sqrt{L + u_n}$$

So our students must first study the sequences and then, knowing how difficult this may be for different values of L or K, devise some kind of worksheet to guide the secondary school student in their discovery. Another example, for the software "MICROCALC", is:

"*Graph any function f. From this graph and using the facilities of the FUNCTION EDITOR graph the following functions:*

a) $g_1(x) = |f(x)|$ b) $g_2(x) = |f(-x)|$ c) $g_3(x) = -|f(x)|$

d) $g_4(x) = -|f(-x)|$ e) $g_5(x) = f(x^2)$ f) $g_6(x) = \left(f(x)\right)^2$

g) $g_7(x) = \sqrt{f(x)}$ h) $g_8(x) = f(\sqrt{x})$

Give some characteristics of each graph and compare them with each other.
Which thoughts this kind of work arises about the use of the computer in the teaching of mathematics?"

Over the years the course changed a little. In the beginning it was more theoretical and philosophical. Now it stresses very much the concrete experiences with the students and the connections with the curriculum the students will have to teach in the future. We discuss less the computer in itself and we practice more with the computer. Now we discuss also what can be done with graphing calculators.

The students react very well to this type of course. They see it is very close to the reality they will have to face afterwards, they see that the discussions include significant mathematics they will teach (and they were not very happy with, when they were secondary school students). The students come regularly to the classes and participate actively. In the final year of their graduation they try to use the computer in their classroom (during the stage) although schools are not very well equipped with computers and the official curriculum is extremely surcharged.

We make a very positive balance of our experience teaching this course.

Annex 1 - Syllabus

General description of the organization and operation of a computer and its peripherals. Notion of hardware, software and courseware. Importance of the software: application, support and system software. General views about programming languages.

Introduction of computers in Schools: characteristics and evolution of the several phases. The influence of computers and Computer Science on Mathematics and on Mathematics Education. Several ways to achieve a "new" mathematics classroom. Role of the teacher

and organization of the classroom in an environment with computers. Uses of the computer in the teaching of Mathematics. Its relation with the teaching of the Applications and Mathematical Modeling.

Programming and the teaching of mathematics: the elaboration of algorithms; problem solving; programming languages.

LOGO: goals, underlying philosophy, consequences of programming in LOGO on the cognitive development of the individual; characteristics of the LOGO programming language.

Discussion of the problem of the assessment of software and its documentation.

View of the renewal of the mathematics curriculum, having in mind the use of technological resources and the new national curriculum for the 2nd and 3rd cycles and secondary education, approved by the Ministry of Education to be implemented in the next years. Impact of the computer in the mathematics curriculum.

Graphing calculators in mathematics teaching.

Exploration of tools: “WORKS” (word processor and spreadsheet), “FUNÇÕES” (graphing software in portuguese), "CABRI-GEOMETRE" (geometry explorations), “SUCESSÕES” (portuguese version of David Smith's "MATHPROGRAM") and “MICROCALC” (portuguese translation), among others, discussing its possible uses in the Mathematics curriculum of Basic and Secondary Education.

Demonstration and informal assessment of some educational software for IBM compatible computers and Macintosh (“TRIGONOMETRIA” (tutorial and tool for trigonometry), “TRINCA-ESPINHAS” (game with prime numbers), “ESPAÇO” (mechanics), “POSIÇÃO” (mechanics), etc).

Activities with the graphing calculator having in mind its possible uses in the mathematics classroom in secondary education.

Study of the LOGO language and its use in the planning of small lesson plans for the teaching of some topics in basic and secondary Mathematics curriculum.

Annex 2 - Sample Examination

Final Exam - 7 / 5 / 94

1st PART - THEORY

1. Make a brief comment, using concrete examples, to the following extract of a text of Prof Bert Waits (Univ. Ohio, EUA) about the reform of mathematics teaching:

"... Students acquire a better comprehension of concepts when these are presented through concrete schemes, numerical or graphical. Technology based on the computer makes this approach available to all students and makes the learning a more active process than the traditional passive process..."

2. Several types of educational software have been called "tools". Which type of tools do you know? Give a characterization of these types and examples of each one.
3. Comment on the importance of a good documentation in educational software.

2nd PART - PRACTICE

1. One of the basic concepts of the new curriculum for secondary education is the concept of *function*. Compare, from the educational point of view, the following two pieces of software you used: "CURVES" and "FUNÇÕES".
2. Describe one concrete activity that can be executed in the mathematics classroom when you have only *one* computer in the classroom. (Refer which piece of software you would use, the goals you had in mind, the role of the teacher, and the strategy you would use).
3. Indicate how the study of the exponential function can benefit form the use of graphing calculators in the mathematics classroom.

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