

USING TECHNOLOGY AS A TOOL IN ABSTRACT ALGEBRA AND CALCULUS:  
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Recently the Department of Mathematical Sciences at Middle Tennessee State University embarked upon the task of restructuring its undergraduate program. An important component of this restructuring was a recommendation for the use of technology to enhance the teaching and learning process. As a result technology was rigorously used in the fall of 1993 in abstract algebra and one section of calculus. The authors of this paper were the instructors of these courses, respectively. In the first part of this paper, Dr. Krishnamani discusses the abstract algebra course, emphasizing changes in the method of teaching abstract algebra, the resulting effect on the student learning pattern, and student concerns. In the second part, Dr. Kimmins emphasizes modifications which were made in her approach to the reformed calculus course as the semester progressed.

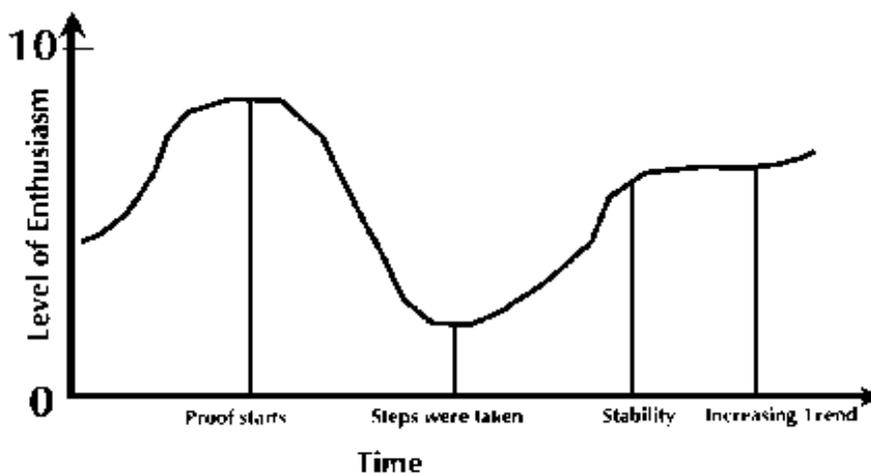
**Using ISETL as a Tool to Learn Abstract Algebra**

A few questions which concern the abstract algebra instructor from time to time are: (i) Are all the students learning? (ii) Could they learn more meaningfully? (iii) Is the teaching method bringing out the best in students? Many different methods were tried to find answers, but none of them were perfect. An opportunity to try the method by Ed Dubinsky of Purdue University was presented at the 1992 International Conference on Technology in Collegiate Mathematics. Following that conference a decision was made to try Dubinsky's method.

The changes: The entire abstract algebra curriculum was redesigned in order to incorporate constructive interactive methods involving computer activities and cooperative learning [4]. The following changes were made by the teacher, and the students were reminded about these changes from time to time. There was a shift in emphasis from grades to learning concepts. There was at least one computer lab assignment per week. There were many open computer lab sessions with supervision. The students were divided into groups. Inter- and intra-group activities were encouraged. There were voluntary and assigned presentations by students. Grades were based on group work, individual presentations, and progress on group and individual tests. There were one-on-one discussions between students and the instructor, both on academic issues and issues pertaining to the changes. These changes resulted in student discoveries. Computer activities enabled the teacher to introduce concepts well in advance of when they could otherwise have been introduced and enabled the students to be more familiar with the ideas, abstract and otherwise. The activities of the students also reflected changes. They diverted from "studying just for tests" to a pattern of continuous learning and having discussions with peers and professors. They became comfortable with making predictions. Some took the role of teachers in their groups as well as in the entire class. Retention improved. Students were proud to present their discoveries.

Observations: Students showed a better understanding of the concept of a group, quantifiers, negation of a proposition, and function. ISETL helped them in logical thinking. They were proficient in applying the tests for subgroups. They understood some finite groups better than the students who had learned abstract algebra in previous semesters. The highlight of using technology was the way students were able to understand cosets and, consequently, predict Lagrange's theorem. They were able to get the sketch of the proof on their own, and then the proof was a mere review. This was a vast improvement over the traditional method.

Student Concerns: The students understandably had many concerns about this new method. These included: i) How much time should be spent on this course? Will I have time to study other subjects? ii) Usually the traditional course has many proofs. Will this course be as good? iii) Will I be able to put in the hard work needed for this course? Is it all worth it? Figure 1 gives an idea of the level of the students' enthusiasm in the course as time progressed.



**Figure 1. Teacher's perception of the students' level of enthusiasm**

Steps to Reduce Anxiety: Several steps were taken to reduce tension. The method was explained in detail. Open discussions were encouraged. Students were allowed to take as much time as they wanted for tests in a more relaxed atmosphere. Some flexibility in the formation of cooperative learning groups was allowed. One day per week was declared students' day; on that day students could talk about their discoveries. The most important step was the homework hotline. This made the students feel secure. Adjustments were made on the high expectations set at the beginning. This was a psychological relief for a few students. Optional problems with extra credit and bonus points were increased. This gave students the opportunity to do their best and solve creative problems. Much time was spent in helping students, especially in the computer lab. A graduate student assisted in the lab. A special week was assigned as abstract algebra awareness week. During this week, the students and the teacher got together as a group and talked informally about abstract algebra. Some students used this week to catch up, others used it for further group discussions. The MAA encourages professional interactions among students and teachers [3]. Activities such as computer sessions and student presentations were the catalysts for these interactions.

Statistics: The class started with 27 students. Two dropped after two days. These students did not want to utilize technology or cooperative learning. One student had a misconception of group work and

dropped after one month. After the first group test, two more students dropped. These two thought that their grades were lowered by their peers in their group. The class continued with the remaining 22 students. Of these, 5 would do well with or without technology, and six were not sure whether technology helped them or not. The real effect of technology was seen in the remaining 11 students who excelled due to ISETL. Without ISETL they would have been the so-called average or below average students in a traditional class. "The quality and the amount of meaningful learning by the average students radically changed" [4].

Performance: Impressive performances by a few students deserve special mention. These talents would have gone unnoticed in a traditional course. Bryan presented a rotation diagram for the group of symmetries of a square and developed specific ISETL code for permutation problems. Teresa Schmidt discovered some cyclic subgroups for a non-cyclic group much earlier than it was taught in class. Mark discussed solutions of the equation  $x^n = e$  in a group with the class. Rebecca had insight concerning the order of an element and that of its inverse in a group well in advance of its discussion in class. Teresa Steinhauer discovered the impossibility of certain sizes for subgroups of a particular group well before Lagrange's theorem was introduced. Julianna, Troy, Pamela, and Kathy developed specific ISETL code for problems involving cyclic subgroups. Faye tried to establish a connection between the length of a cycle and the parity of the permutation.

Recognition: Extra recognition, besides grades, was given. Students presented their own results to the admiration of others. A few students assumed leadership roles which they would not have normally done. Each student's test was personalized in appearance. Each student felt that his or her answers were important. Discussion of goals and performance was done on an individual basis. A special certificate was issued to students for learning ISETL.

In 1993, the U.S. Department of Education launched an algebra initiative in an attempt to establish a coherent approach to algebra education and algebra reform at every level of education, Educators at the conference at which this initiative was launched were encouraged to consider the use of technology in abstract algebra, and ISETL was cited as one of the well-suited computational environments [1]. New ideas and solutions are required to face the challenges encountered in the use of technology, University teachers need to try technology and do research in different methods of teaching. This will enable us to develop effective ways to use technology to help students learn algebra, "Undergraduate mathematics is the lynchpin of mathematical education. No reform of education is possible unless it begins with revitalization of mathematics in both curriculum and teaching style" [2].

### **A Description of a First Attempt at Teaching a Reformed Calculus Course**

By way of background information, in the academic years 1991-92 and 1992-93, a colleague taught one section of the calculus sequence using the Duke materials. I was asked to teach the Project CALC section in the fall of 1993 in an effort to get more faculty involved in teaching reformed calculus. All other sections used the Finney-Thomas text. My attitude toward teaching the reformed section was influenced by Heather, who in my traditional calculus course in the spring of 1993 asked "What is a derivative?" two-thirds of the way into the semester. I was willing to try anything; I felt that I could not have less success in achieving student understanding of the fundamental concepts of calculus than I had in the spring of 1993. Students did not know in advance that they were enrolling in a non-traditional calculus course, but were told on the first day of class that their section would be taught as a laboratory course. If students reacted negatively, they were allowed to transfer to another section. Approximately 15% did transfer.

Initially I taught using the suggested schedule of Smith and Moore which appears in the instructor's manual accompanying the Duke materials. The suggested schedule includes recommendations for the focus of each class period and accompanying classroom activities. Likewise, in keeping with the philosophy of the originators of Project CALC, I initially depended on lecturing very little; students were expected to digest the text. Students were told that in lieu of lectures, class time would be spent in mini-lectures when needed, classroom demonstrations and activities, small-group projects, computer laboratories, and answering student questions. In keeping with the spirit of Project CALC (Calculus as a Laboratory Course), the computer lab component was to motivate the entire course,

As the semester progressed, it became increasingly apparent that students were not reading and/or understanding the text and were able to complete the labs only by badgering me for help. I suspected students were not reading the text when it became apparent by their questions that they were not reading lab instructions. I decided that some modification had to be made several weeks into the semester when we completed the lab "Falling Bodies with Air Resistance." In this lab, as in a previous lab, students used Euler's method. After completing the "falling bodies" lab, a majority of students still did not seem to understand Euler's method. Thus, I began introducing material in class as I had previously done in Heather's class, in a lecture-type format (although I have learned that when I consider myself lecturing, many do not really consider it to be lecturing because of my effort to weave student interaction into the discussion). The labs were then used to reinforce this material which had been previously introduced in class and allow students to experience rich applications of this material. As a result of this change, it was impossible to assign all of the labs.

I found myself occasionally writing a lab. The students seemed to particularly enjoy and benefit from a lab on infinite series which I wrote during the spring of 1994 for my second semester Project CALC course. In this lab students gathered numerical evidence for convergence or divergence of common series. For some of the series, students were asked to write the general term in the sequence of partial sums and thus verify their conjectures formed from examining the numerical evidence. An interesting footnote is that one lab pair became really interested in finding the general term in the sequence of partial sums for the  $p$ -series with  $p=2$  and thus the exact sum. For obvious reasons students were not asked to do this in the lab. When I saw this pair was so interested, I assigned this as a bonus project. The Wednesday before Thanksgiving this pair was in my office reporting on their efforts, which included library research. They finally completed the project, but never wrote it up for credit. It was extremely refreshing to see students investigate mathematics for the sheer joy of it!

Students worked in pairs on computer labs. The pairs were formed by the instructor. Two lab pairs were placed together by the instructor to form cooperative learning groups for other activities such as projects. These pairings were changed at the instructor's discretion during the semester. There was some initial trepidation by students about cooperative learning. I recall one student on the first day of class telling me that he would give it a try, but it really concerned him that his grade would be based on work that was not solely his own. I was pleased when later in the semester I specified that an assignment had to be completed individually, and he responded with a comment of the type "Can't we work together?"

I was not experienced in directing cooperative learning and very quickly learned that my attempts at structuring group work were effective when interesting multi-step problems were the object of the cooperation, but ineffective when, for example, routine homework discussion was the object of the cooperation. I found the projects in the Project CALC materials to be ideal for cooperative learning groups to tackle. To hear students discuss mathematics as related to their attempts at solving the "air-traffic control" problem, for example, was refreshing. However, I was amazed and appalled at the attitude many students had toward their lab partners. It was not uncommon for one of a pair to miss class

for a lab, leaving their partner without a diskette on which vital previous work had been done. Because of the comments of a few students in their evaluations of the first semester, students were given the option of working cooperatively or individually on computer labs during the second semester; approximately half chose to work individually.

Student comments on course evaluations revealed that students often did not see the connection between the labs and the course. Students wrote that "some of the labs have not seemed to be related to the material in the book" and the labs "seemed to stray from the stuff in class." One student complained bitterly in frustration when she was doing the lab on compound interest that "this is not calculus."

One student in his course evaluation of my second semester reformed calculus course wrote: "The addition of the computer laboratory is especially helpful because I can see some real application to some principles that I wouldn't see elsewhere. It also is good for a change of pace from the drudgery of simply 250 minutes of lecture weekly." However, there was some frustration with the use of computers both semesters. One student indicated that she thought there should be a computer prerequisite for the course; another remarked, "Having no background and experience with computers, I find I get frustrated easily when MCAD doesn't work properly." In course evaluations given in conjunction with the final exam the first semester, of the 13 students who completed the evaluation (two students who took the final did not complete the evaluation), 7 students "strongly" agreed with the statement, "I am glad we had a computer lab component to this course." Four students "somewhat" agreed, and two students "somewhat" disagreed. In response to the statement, "I feel that I benefited from the computer lab component," eight "strongly" agreed, four "somewhat" agreed, and one "somewhat" disagreed. It is interesting to note that one of the students who "somewhat" agreed with the statement, "I am glad we had a computer lab component to this course" and one of the students who "somewhat" disagreed with this statement strongly agreed with the statement, "I feel that I benefited from the computer lab component."

As the first semester progressed I found myself supplementing the excellent problems in the Project CALC text, The Calculus Reader, with additional problems of a more routine computational nature. The two tests, in addition to the final exam, were of both a conceptual and a computational nature. Fifty percent of the semester grade was based on labs and projects done cooperatively, but students were required to score 100% on a computational gateway test on differentiation in order to exit the course. All but one student who took the final exam successfully completed this requirement, although several students required several attempts.

Students were definitely more engaged, active, and interested as a result of the nature of the Project CALC course content, labs, and projects than the class in which Heather sat in the spring of 1993. In addition, through the use of computers, students had more exposure to the numeric and graphic interpretations of the fundamental concepts of calculus and investigated rich mathematical applications. In my first attempt at the reformed course, calculus was still a filter and not a pump, but boredom and inactivity seemed to disappear.

### References

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