GROUP RESEARCH PROJECT AND COOPERATIVE LEARNING IN STANDARD CALCULUS

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After conducting several experimental studies, using a cooperative learning approach, I have decided to incorporate this approach in teaching second semester standard calculus by adding a new component consisting of ten **Group Research Projects**. The main objectives were to not only give the students more responsibility for their own learning and discovering of mathematics, but to learn how to do the proofs as well.

On the first day of the class students were divided into heterogeneous groups of five members each, considering past performance, sex and race. I chose one student in each group to be the leader of his/her group, taking the above factors into account.

The leader's role and responsibilities, guidelines for groups' outside classroom activities and monitoring the groups were the same as the previous experimental studies (see **[1]** for more details).

The instructional style was based on a brief lecture on each topic. That is, I provided enough (but not complete) information in class discussions so that the groups could work effectively on specific problems. It should be noted that I used a standard calculus textbook by Philip Gillet (third edition).

A group research project was another component of my instructional plan. As the students were getting exposed to new materials, a relevant research project was given to all groups (on a regular basis) to complete in one or two weeks, depending on the level of the difficulty of the projects. The typical projects[1] were:

1. A Continuous Additive Function:

In this project you will learn about additive functions and discover that continuous additive functions have a very special form.

Definition. Let f be a function whose domain is the set of all real numbers. Then f is *additive* if $f(x + y) = f(x) + f(y)$ for all real numbers x and y.

a. Give an example of an additive function and show that it is additive. Give an

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[1] Most of the projects were selected from **[2]**.

example of a function that is not additive and show that it is not additive.

b. Suppose that f is an additive function and m is any real number. Define a new function g by the formula $g(x) = f(x) - mx$. Show that g is an additive function.

c. Let $m = f(1)$ and show that the function g, defined in part (**b**) above, has the property that $g(x + 1) = g(x)$ for all x.

2. A Continuous Affine Function:

In this Project you will learn about affine functions and discover that continuous affine functions have a very special form.

Definition. Let f be a function whose domain is the set of all real numbers. Then f is *affine* if $f(x - y + z) = f(x) - f(y) + f(z)$ for all real numbers x,y, and z.

a. Give an example of an affine function and show that it is affine. Give an example of a function that is not affine and show that it is not affine.

Definition. Let g be a function whose domain is the set of all real numbers. Then g is *additive* if $g(x + y) = g(x) + g(y)$ for all real numbers x and y.

b. Suppose that f is an affine function and m is any real number. Define a new function g by the formula $g(x) = f(x) - (mx + f(0))$. Show that g is an additive function.

c. Let $m = f(1) - f(0)$ and show that the function g defined in part (**b**) above has the property that $g(x + 1) = g(x)$ for all x.

Definition. A function f is *bounded* on a close interval $[a, b]$ if there is a real number B such that $|f(x)| \le B$ for all x in $[a, b]$. A function f is bounded if there is a real number B such that $|f(x)| \le B$ for all x.

d. Suppose that f is an affine number that is bounded on $[a, b]$ and g is defined by $g(x) = f(x) - (mx + b)$ where $m = f(1) - f(0)$ and $b = f(0)$. Show that g is bounded.

3. The number *e* is Irrational:

Fill in the missing steps in this sketch of a proof that e is irrational. The proof is by contradiction.

Proof. Suppose to the contrary that e is rational. Then $e = p/q$ where p and q are positive integers. Let

$$M = q!\left\{e - \sum_{k=0}^{q} \frac{1}{k!}\right\}.$$

Explain why M is a positive integer. Explain why

$$M = q! \sum_{k=q+1}^{\infty} \frac{1}{k!} = \sum_{k=q+1}^{\infty} \frac{q!}{k!}.$$

Compare this favorably with a geometric series to show that $M < 1/q$. Explain exactly how this completes the proof that e is irrational.

4. Power Series with Positive and Negative Exponents:

A Maclaurin series is a series of the form $\Sigma\, a_n x^n$, for $n = 0$ to ∞. A familiar example is the geometric series $\Sigma\, x^n$, for $n = 0$ to ∞, which converges to $1/(1 - x)$ when $-1 < x < 1$. The function $f(x) = 1 / (1 - x)$ may also be written as a sum of negative powers of x. A little Algebra shows that

$$\frac{1}{1-x} = -\frac{1}{x}\left[\frac{1}{1-\frac{1}{x}}\right] = -\frac{1}{x}\sum_{n=0}^{\infty} x^{-n} = -\sum_{n=1}^{\infty} x^{-n}. \qquad (*)$$

a. For what values of x are the equalities in (*****) all valid?

b. Find a Maclaurin series for $f(x) = 1 / (x - 2)$. What is the region of convergence? Now find a series of negative powers of x for the same function. What is the region of convergence of this series?

Sometimes a series can have both positive and negative powers of x. Such a series,

$$\sum_{n=0}^{\infty} a_n x^n + \sum_{n=1}^{\infty} b_n x^{-n},$$

is said to converge if and only if both $\Sigma\, a_n x^n$, for $n = 0$ to ∞, and $\Sigma\, b_n x^{-n}$, for $n = 1$ to ∞, converge.

c. Use partial fractions to write the rational function, $R(x) = 1/(x^2 - 3x + 2)$, as a sum of fractions with linear denominators.

d. Each of the two fractions with linear denominators you found in part (**c**) will have two series, one with only nonnegative powers of x and one with only negative powers of x. This will give you four possibilities for a series for $R(x)$. Find those series and their respective regions of convergence.

5. **Gamma Function:**

The *gamma* function is defined for all $x \geq 1$ by the rule

$$\Gamma(x) = \int_0^{\infty} t^{x-1} e^{-t} dt$$

a. Show that $\Gamma(1) = 1$ and $\Gamma(2) = 1$.

b. Use integration by parts to prove that

$$\Gamma(x+1) = x\Gamma(x) \qquad \text{for all } x \geq 1$$

c. If we agree to the convention that $0! = 1$, explain why $\Gamma(n) = (n - 1)!$ for every positive integer n.
(Thus the gamma function is an ordinary factorial when $x = n$. It is also defined for values like $x = 3/2$ or $x = \pi$, when the factorial notation does not make sense).

The *Laplace Transform* of a function f is the function L defined by

$$L(x) = \int_0^{\infty} e^{-xt} f(t)\, dt.$$

Find the Laplace Transform of each of the following functions.

d. $f(t) = 1$

e. $f(t) = e^t$

f. $f(t) = \sin t$

At first, the students had no difficulty completing part **a** of their first project (number **1** above). But for parts **b** and **c** they thought that considering and verifying a specific function would be a sufficient proof. At this time, I used a portion of the class time to explain to them, that if you want to **disprove** a statement, one example - called a **counterexample** - will do it. But if we want to prove that **a mathematical statement is true**, verification of millions of examples will not lead to a proof. Following my brief lecture on this matter, the groups did very well on their second project (number **2** above).

Students were usually given a portion of or at times the whole class period to work on their homework assignments or group research projects. My role was to circulate and provide help or some hints as needed. The students successfully completed nine out of ten projected **Group Research Projects**. Some of the projects included optional sections of the textbook or a problem from the exercise section (such as number **5** above). They often explored new ideas (not discussed in the textbook), which although important, could not be covered due to time constraints.

The student evaluation scheme was based on two factors: **Individual Evaluation** - midterm and final exams (**50%** weight), and **Group Evaluation** - group homework assignments

Gave me the willingness to persevere when solutions are not immediate.	85	8	7
Lead me to attribute slow progress in finding answers to not using the right strategy rather than not being competent.	79	16	5

As we observe, the overall evaluation indicates that most students found the group learning strategy to be a better way of learning mathematics.

CONCLUSION

The outcome of this experimental study can be summarized as follows:

1. The collaborative learning approach leads the students to be actively involved in working out specific problems and exploring mathematical concepts and ideas for themselves rather than being solely a recipient of knowledge.

2. Students are given more responsibility for their own learning and discovery of mathematics. And students are willing to accept such responsibility.

3. The study showed that students at this level are capable of learning how to do proofs via Group Research Projects - an objective which is not expected or at least is not actively pursued in the teaching of regular calculus courses.

4. More material than what was projected in their syllabus has been covered, and new concepts and ideas were explored via group research projects.

5. This approach does provide the opportunity for students to communicate mathematics by emphasizing reading, writing, and speaking skills.

6. It does tend to promote high achievement.

7. Students learn to cooperate with other members of their groups.

8. Students learn and practice group decision-making which is an important element in today's marketplace.

9. The teacher-student relationship can be closer than in a traditional approach.

10. The classroom atmosphere is friendly and less formal so that students feel free to ask questions, get involved in group discussions, and respond to questions without having a fear of being wrong.

11. Close contact with students during their group activities in or outside the classroom provided the opportunity to see how each student was thinking about and approaching the solution of a particular math problem.

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